Problem E1- Solution

Heat Conduction in a Copper Rod (10 points)

0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

Part A: The short copper rod (3.9 points)

A.1 (0.2 pt)

$R_{en} = 110.11 \pm 0.01\ \Omega$

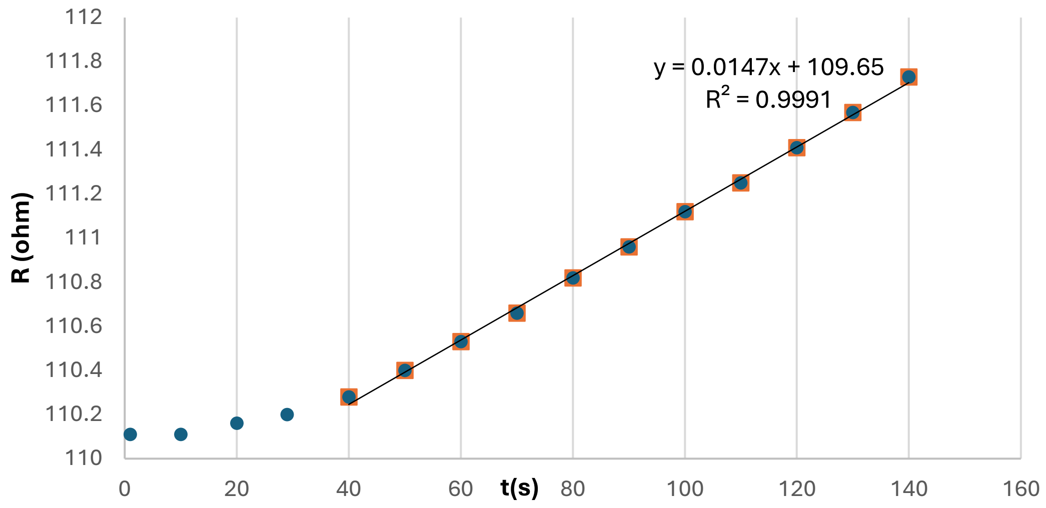
$$\theta_{en} = \frac{R - R_0}{R_0 \alpha}$$

$\theta_{en} = 25.87 \pm 0.03\ ℃$

A.2 (0.5 pt)

n	$R(\Omega)$	$t(s)$
1	110.11	1
2	110.11	10
3	110.15	20
4	110.18	30
5	110.26	40
6	110.4	50
7	110.53	60
8	110.66	70
9	110.82	80
10	110.96	90
11	111.12	100
12	111.25	110
13	111.41	120
14	111.57	130
15	111.73	140

A.3 (0.8 pt)

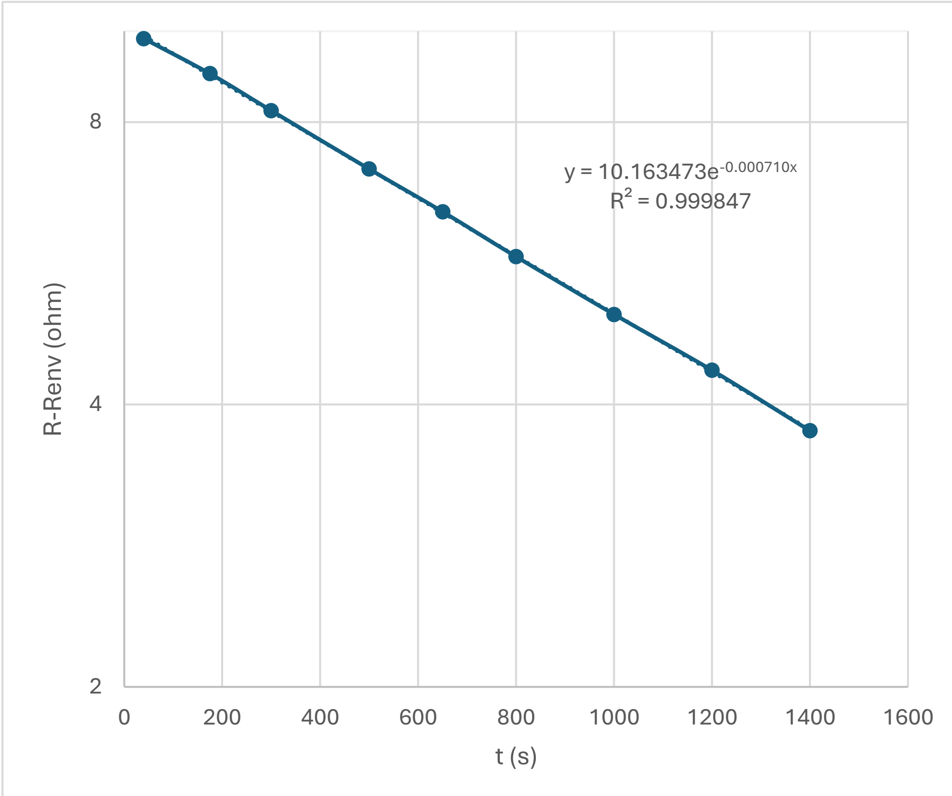


$\Delta R/\Delta t$	$0.0147\ \Omega/s$	$\frac{\Delta\theta}{\Delta t} = \frac{1}{R_0\alpha}\frac{\Delta R}{\Delta t}$
$\Delta\theta/\Delta t$	$0.03761\ {}^\circ C/s$	
C_s	$52 \pm 2\ \ J/{}^\circ C$	$C_s = \frac{P_1}{\frac{\Delta\theta}{\Delta t}}\quad, \Delta C_s \approx C_s\frac{\Delta P_1}{P_1}$

A.4 (0.5pt)

n	$R(\Omega)$	$t(s)$	$(R - R_{en})(\Omega)$
1	119.98	40	9.82
2	119.17	175	9.01
3	118.39	300	8.23
4	117.29	500	7.13
5	116.58	650	6.42
6	115.91	800	5.75
7	115.15	1000	4.99
8	114.51	1200	4.35
9	113.91	1400	3.75
10	113.40	1600	3.24

A.5 (0.7 pt)

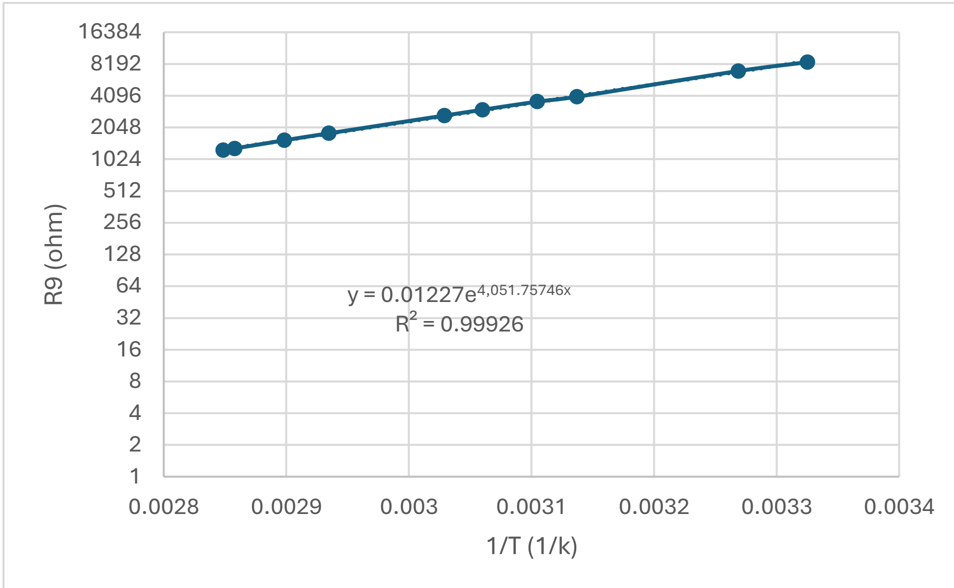


$\gamma = (710 \pm 3) \times 10^{-6}\ s^{-1}$

A.6 (0.5 pt)

n	$R(\Omega)$	$R_9(\Omega)$	$T(k)$	$\frac{1}{T}\left(\frac{1}{k}\right)$
1	8528	110.78	300.733	0.003325
2	7020	112.81	305.9272	0.003269
3	4005	117.83	318.772	0.003137
4	3601	119.13	322.0984	0.003105
5	3014	120.96	326.7808	0.00306
6	2658	122.27	330.1328	0.003029
7	1803	126.42	340.7515	0.002935
8	1547	128.09	345.0245	0.002898
9	1296	130.00	349.9117	0.002858
10	1246	130.45	351.0631	0.002848

A.7 (0.7 pt)



$E_g = 0.698 \pm 0.007\ eV$

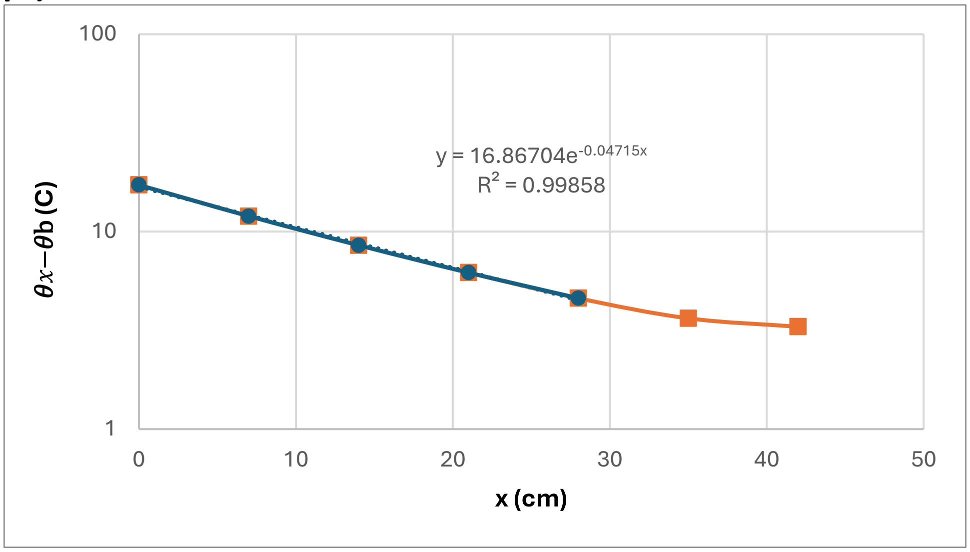
Part B: The long copper rod (4.1 points)

B.1 and B.5

$\theta_b = \ 24.41°\mathrm{C}$

B.1 (0.4pt)				B.5 (0.4pt)	
n	$x(cm)$	$\theta_x(°\mathrm{C})$	$\theta_x - \theta_\mathrm{b}(°\mathrm{C})$	$B^{(1)}e^{\lambda^{(0)}x}(°\mathrm{C})$	$\theta'_x - \theta_\mathrm{b}(°\mathrm{C})$
1	*0*	44.61	17.23	0.27	16.96
2	*7*	39.37	11.99	0.37	11.62
3	*14*	35.92	8.54	0.51	8.03
4	*21*	33.58	6.20	0.72	5.48
5	*28*	31.98	4.60	1.00	3.60
6	*35*	31.02	3.64	1.39	2.25
7	*42*	30.68	3.30	1.93	1.37

B.2 (0.4 pt)



B.3 (0.6 pt)

$A^{(0)} = 16.87\ ℃$

$\lambda^{(0)} = 0.047 \pm\ 0.005\ \left(\frac{1}{cm}\right)$

B.4 (0.4 pt)

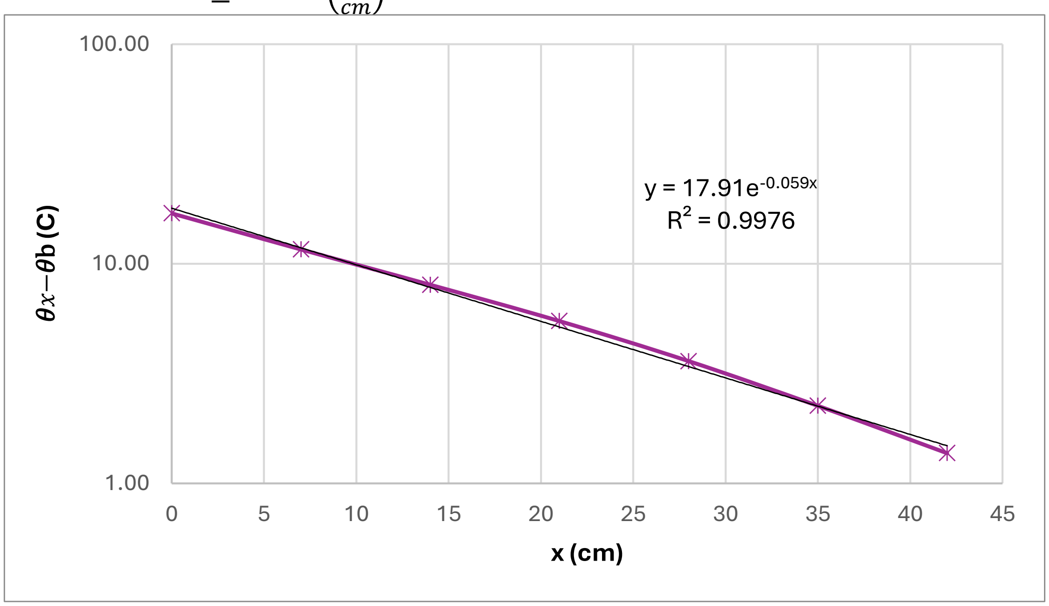
$B = Ae^{-2\lambda d}$

$B^{(1)} = 0.266℃$

B.6 (1.0 pt)

$A^{(1)} = 17.91℃$

$\lambda^{(1)} = 0.059 \pm\ 0.002\ \left(\frac{1}{cm}\right)$



B.7 (0.9 pt)

$$P_2 = \int_{-0.5cm}^{42.5cm} 2\pi r h(\theta_x - \theta_b)dx \quad \text{or} \quad \int_0^d 2\pi r h(\theta_x - \theta_b)dx = \frac{2\pi r h A}{\lambda}(1 - e^{-2\lambda d})$$

$$h = \frac{P_2 \lambda}{2\pi r A(1 - e^{-2\lambda d})}, \; k = \frac{2h}{\lambda^2 r}, P_2 = 4.5\,W$$

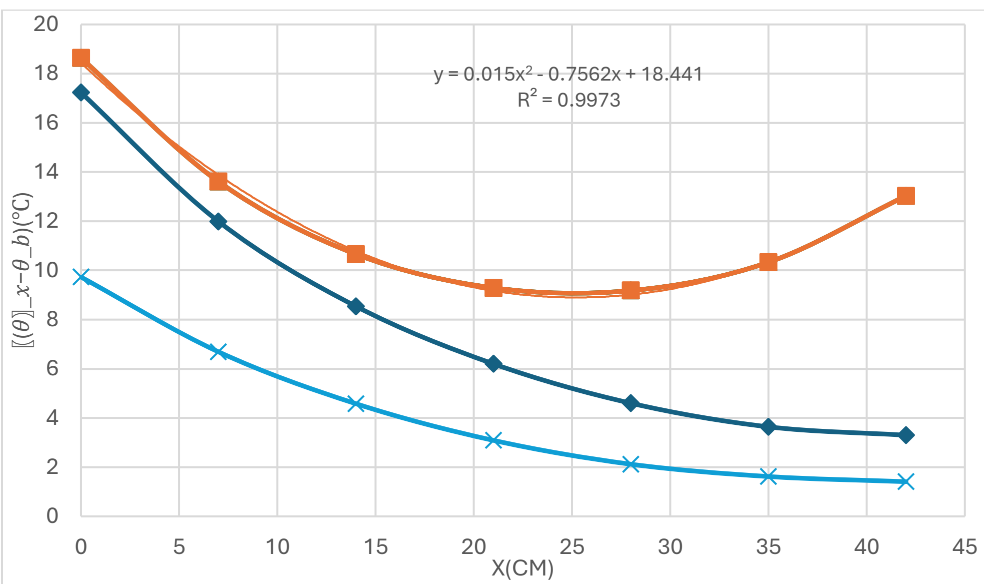
$$\lambda = 0.053 \pm 0.008 \left(\frac{1}{cm}\right), \; h = 0.0037 \pm 0.0003 \frac{W}{k\,cm^2}, k = 3.9 \pm 0.3 \frac{W}{k\,cm}$$

C.1 (0.4 pt)

$\theta_b = 27.38$°C

n	$x(cm)$	θ_x(°C)	$(\theta_x - \theta_b)$(°C)	B-1	C-B	Reversed order
1	0	46.02	18.64	17.23	1.41	9.73
2	7	40.99	13.61	11.99	1.62	6.69
3	14	38.04	10.66	8.54	2.12	4.58
4	21	36.67	9.29	6.2	3.09	3.09
5	28	36.56	9.18	4.6	4.58	2.12
6	35	37.71	10.33	3.64	6.69	1.62
7	42	40.41	13.03	3.3	9.73	1.41

C.2 (0.6 pt)

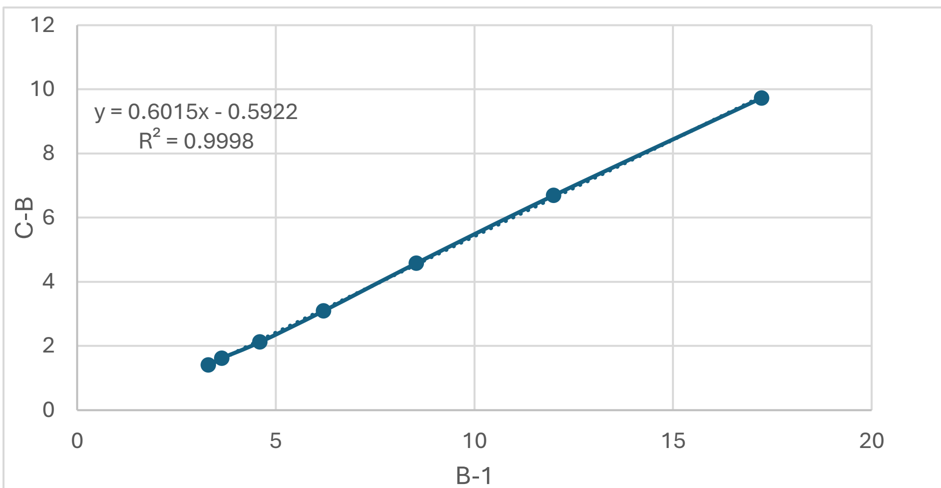


(The blue plots are not necessary to draw)

C.3 (1.0 pt)

1) $\frac{P_3}{P_2} = \frac{\frac{d\theta}{dx}_{x=42cm}}{\frac{d\theta}{dx}_{x=0cm}} = \frac{\frac{\Delta\theta}{\Delta x}_{x=42cm}}{\frac{\Delta\theta}{\Delta x}_{x=0cm}} = \frac{\theta_7 - \theta_6}{\theta_0 - \theta_1}$
2) $\frac{P_3}{P_2} = \frac{\frac{d\theta}{dx}_{x=42cm}}{\frac{d\theta}{dx}_{x=0cm}} = \frac{\sinh(\lambda(42 - x_0))}{\sinh(\lambda x_0)}$
3) calculation by integral method or sigma in several parts

4) Slope ((C-B) _ B)



$$P_3 = 0.60\ P_2 \pm 0.01\ P_2$$

Problem E1-Marking scheme Heat Conduction in a Copper Rod (10 points)

Part A: The short copper rod (3.9 points)

A.1 (0.2 pt)	0.05 points if $112 > R_{env} > 108\ \Omega$ $\Delta R_{env} = \pm 0.01\Omega$ (0.05 pt)	0.1 pt
	This value should be calculated from this equation $\theta_{env} = \frac{R-R_0}{R_0\alpha}$. Calculation of θ_{env} using the obtained results (0.05 pt) Calculation of the uncertainty $\Delta\theta_{env} = \pm 0.03$ (0.05 pt)	0.1 pt

A.2 (0.5 pt)	Missing measurement points less than 15 time (-0.04 each) For each interval out of range 8-12 seconds (-0.02 each) Wrong measurement of resistance (-0.04 each)	0.5 pt

A.3 (0.8 pt)	Graph (max 0.4) Data points are properly plotted. (0.3) Too small (-0.1) Missing of the quantities or units (-0.1) Line to determine the slope (0.1)	0.4 pt
	Reading of $\frac{\Delta R}{\Delta t}$ from the graph or calculator. $\frac{\Delta R}{\Delta t}$ is in range $0.0138 \leqq \frac{\Delta R}{\Delta t} \leqq 0.0165\ \frac{\Omega}{s}$ (0.1) Missing or incorrect units (-0.05)	0.1 pt
	Calculation of $\frac{\Delta\theta}{\Delta t} = \frac{1}{R_0\alpha}\frac{\Delta R}{\Delta t}$ using the obtained result.	0.1 pt
	47 $\leqq C_s \leqq$ 52 J/°C (0.1 pt) , 44 $\leqq C_s \leqq$ 55 J/°C (0.05 pt) . Missing or incorrect units (-0.05)	0.1 pt
	Correct ΔC_s formula (0.05 pt) Calculation of the uncertainty $\Delta C_s = \pm 1\ or\ 2$ J/°C (0.05 pt)	0.1 pt

A.4 (0.5 pt)	Missing measurement points less than 10 time (-0.05 each)	0.5 pt

A.5 (0.7 pt)	Graph (max 0.4) Data points are properly plotted (0.3). Too small (-0.1) Missing of the axes or quantities or units (-0.1) Missing points (-0.04 each) Line to determine the slope (0.1)	0.4 pt
	$6.5 \times 10^{-4} \leqq \gamma \leqq 7.5 \times 10^{-4}$ 1/s (0.2 pt),$6\times 10^{-4} \leqq \gamma \leqq 8\times 10^{-4}$ 1/s (0.1 pt) Missing or incorrect units (-0.05)	0.2 pt
	Calculation of the uncertainty $\Delta\gamma$ (0.05 pt), Correct $\Delta\gamma$ formula (0.05 pt)	0.1 pt

A.6 (0.5 pt)	Missing measurement points less than 7 time (-0.05 each) For not covering a resistance range of 5 Ω, each 0.5 Ω (-0.02) Inconvenient distribution of data points (up to -0.1)	0.5 pt

A.7 (0.7 pt)	Graph (max 0.4) Data points are properly plotted (0.3). Too small (-0.1) Missing of the axes or quantities or units (-0.1) Missing points (-0.04 each) Line to determine the slope (0.1)	0.4 pt
	$0.68 \leqq E_g \leqq 0.72\ eV$ (0.2), $0.66 \leqq E_g \leqq 0.74\ eV$ (0.1) Missing or incorrect units (-0.05)	0.2 pt
	Calculation of the uncertainty ΔE_g	0.1 pt

Part B: The long copper rod (4.1 points)

B.1 (0.4 pt)	Missing measurement points less than 7 time (-0.05 each) $22 > \theta_1 - \theta_b > 15$°C and $\theta_7 - \theta_b > 3$°C (0.1)	0.4 pt

B.2 (0.4 pt)	Graph (max 0.4) Data points are properly plotted (0.3). Too small (-0.1) Missing of the axes or quantities or units (-0.1) Missing points (-0.05 each) Line to determine the slope (0.1)	0.4 pt

B.3 (0.6 pt)	Calculation of $A^{(0)}$ using the obtained results. Missing or incorrect units(-0.05)	0.2 pt
	$0.046 \leqq \lambda^{(0)} \leqq 0.051 \left(\frac{1}{cm}\right)$ (0.3), $0.044 \leqq \lambda^{(0)} \leqq 0.053 \left(\frac{1}{cm}\right)$ (0.2) Missing or incorrect units (-0.05)	0.3 pt
	Calculation of the uncertainty $\Delta\lambda^{(0)}$	0.1 pt

B.4 (0.4 pt)	$B = Ae^{-2\lambda d}$ Correct equation	0.2 pt
	Calculation of B(1) using the obtained results	0.2 pt

B.5(0.4)	Incorrect calculation (-0.05 each)	0.4 pt

B.6 (1.0 pt)	Graph (max 0.4) Data points are properly plotted (0.3). Too small (-0.1) Missing of the axes or quantities or units (-0.1) Missing points (-0.05 each) Line to determine the slope (0.1)	0.4 pt
	Calculation of $A^{(1)}$ using the obtained results. Missing or incorrect units (-0.05)	0.2 pt
	$0.054 \leqq \lambda^{(1)} \leqq 0.063 \left(\frac{1}{cm}\right)$ (0.3), $0.051 \leqq \lambda^{(1)} \leqq 0.068 \left(\frac{1}{cm}\right)$ (0.2) Missing or incorrect units (-0.05)	0.3 pt
	Calculation of the uncertainty $\Delta\lambda^{(1)}$	0.1 pt

B.7 (0.9 pt)	Calculation of the uncertainty λ	0.2 pt
	Calculation of the uncertainty $\Delta\,\lambda$	0.1 pt
	Correct formula for h (0.1), Calculation of h using the obtained results (0.1)	0.2 pt
	Calculation of the uncertainty Δh	0.1 pt
	Correct formula for k (0.1), Calculation of k using the obtained results (0.1)	0.2 pt
	Calculation of the uncertainty Δk	0.1 pt

Part C: Measuring the unknown power (2.0 points)

C.1 (0.4 pt)	Missing measurement points less than 7 time (-0.05 each)	0.3 pt
	$25 > \theta_1 - \theta_b > 15℃ \; and \; \theta_7 - \theta_b > 10℃$ (0.1)	0.1 pt

C.2 (0.6 pt)	Graph (max 0.4) Data points are properly plotted (0.3). Too small (-0.1) Missing of the axes or quantities or units (-0.1) Missing points (-0.05 each) Plot a curve trough the data (0.1)	0.4 pt
	Find x_0 using the Quad mod on the calculator or find x_0 from graph (0.1) $24.5 \leqq x_0 \leqq 25.5$ cm (0.1), $24 \leqq x_0 \leqq 26$ cm (0.05)	0.2 pt

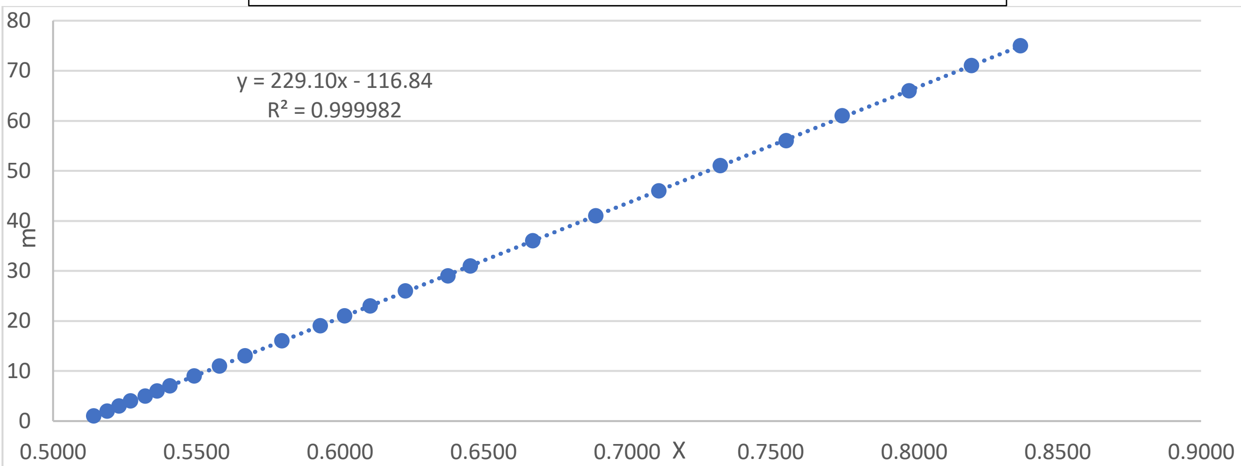
C.3 (1.0 pt)	Calculation by subtracting the results of part C and B and calculating the slop of the line (0.6) Calculation by sinh and x_0 (0.4) Calculation by the integral (0.4) Calculate the slope of 2 points (0.2) Another method (?)	0.6 pt
	$0.59 \leqq \frac{P_3}{P_2} \leqq 0.61$ (0.3), $0.58 \leqq \frac{P_3}{P_2} \leqq 0.62$ (0.2), $0.57 \leqq \frac{P_3}{P_2} \leqq 0.63$ (0.1) Missing or incorrect units (-0.05) Calculation of the uncertainty P_3 (0.1)	0.4 pt

Problem E2- Solution

A.1			
number	m	θ_m (degrees)	$\sqrt{n^2 - \sin^2\theta_m} - \cos\theta_m$
1	1	9.00	0.5142
2	2	13.00	0.5188
3	3	15.75	0.5229
4	4	18.00	0.5270
5	5	20.50	0.5322
6	6	22.25	0.5362
7	7	24.00	0.5406
8	9	27.00	0.5491
9	11	29.75	0.5579
10	13	32.25	0.5668
11	16	35.50	0.5798
12	19	38.50	0.5931
13	21	40.25	0.6015
14	23	42.00	0.6105
15	26	44.25	0.6228
16	29	46.75	0.6375
17	31	48.00	0.6453
18	36	51.25	0.6671
19	41	54.25	0.6891
20	46	57.00	0.7110
21	51	59.50	0.7325
22	56	62.00	0.7555
23	61	64.00	0.7750
24	66	66.25	0.7982
25	71	68.25	0.8200
26	75	69.75	0.8371

S2-2

A.2



A.3

B = 229.1

A = -116.8

A.4

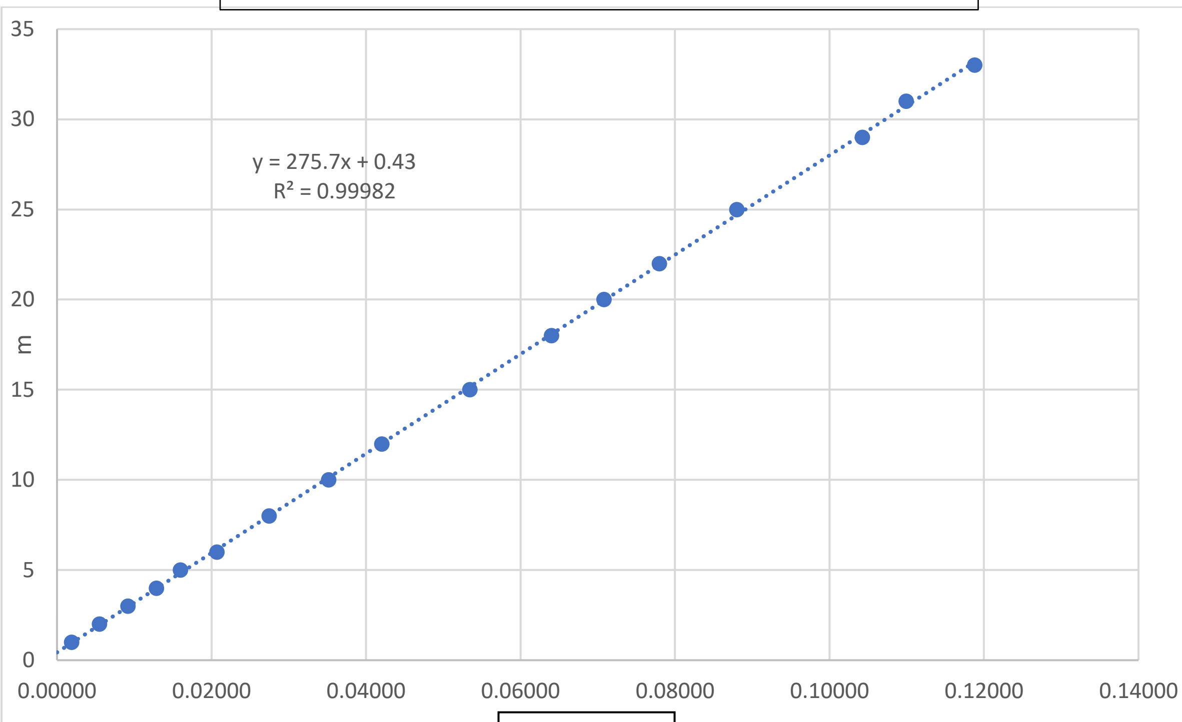
h = $(148.9 \pm 0.1)\ \mu m$

B.1			
number	m	θ_m (degrees)	${\theta_m}^2$ (rad^2)
1	1	2.50	0.00190
2	2	4.25	0.00550
3	3	5.50	0.00921
4	4	6.50	0.01287
5	5	7.25	0.01601
6	6	8.25	0.02073
7	8	9.50	0.02749
8	10	10.75	0.03520
9	12	11.75	0.04206
10	15	13.25	0.05348
11	18	14.50	0.06405
12	20	15.25	0.07084
13	22	16.00	0.07798
14	25	17.00	0.08803
15	29	18.50	0.10426
16	31	19.00	0.10997
17	33	19.75	0.11882

B.2

$$m = \frac{H}{2\lambda}\left(1 - \frac{1}{n}\right){\theta_m}^2$$

B.3



$\theta_m^2\ (rad^2)$

B.4

B = 275.7

A =0.43

B.5

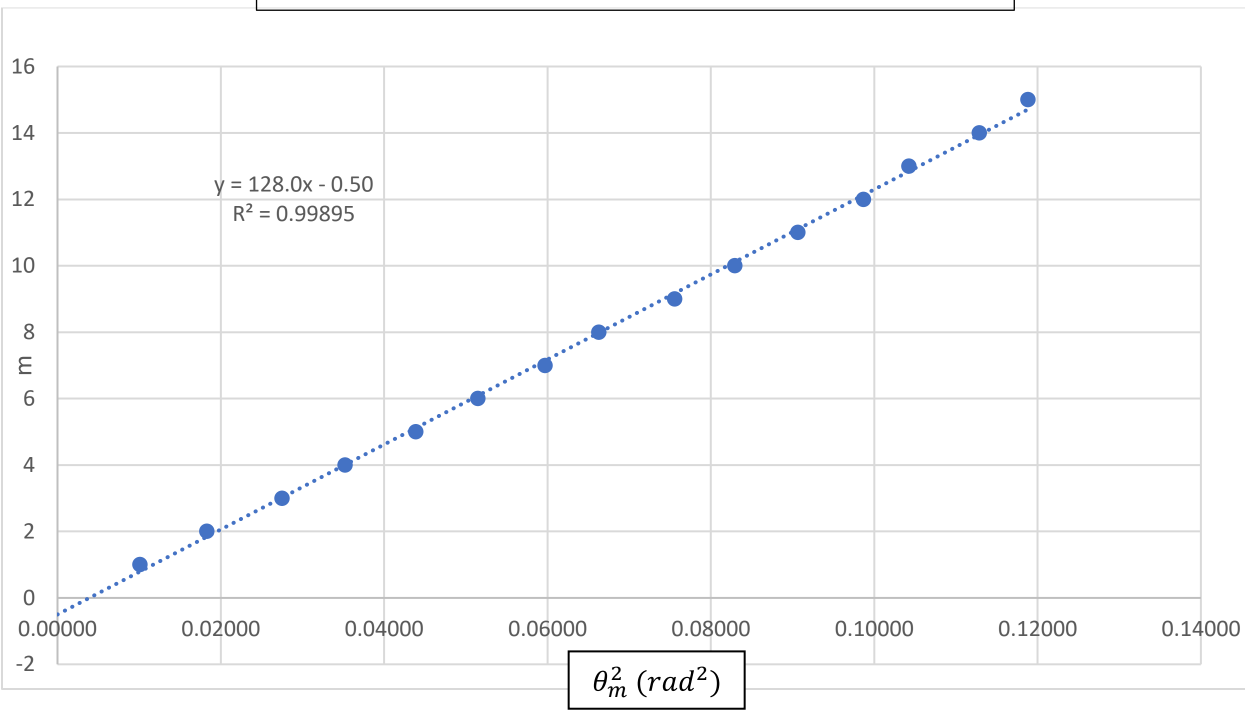
H = $(1.061 \pm 0.004)mm$

C.1			
number	m	θ_m (degree)	${\theta_m}^2$ (rad^2)
1	1	5.75	0.01007
2	2	7.75	0.01830
3	3	9.50	0.02749
4	4	10.75	0.03520
5	5	12.00	0.04386
6	6	13.00	0.05148
7	7	14.00	0.05971
8	8	14.75	0.06627
9	9	15.75	0.07556
10	10	16.50	0.08293
11	11	17.25	0.09064
12	12	18.00	0.09870
13	13	18.50	0.10426
14	14	19.25	0.11288
15	15	19.75	0.11882

C.2

$$m = \frac{H}{2\lambda}\left(N - \frac{N^2}{n}\right){\theta_m}^2$$

C.3



C.4

B = 128.0

A = -0.50

C.5

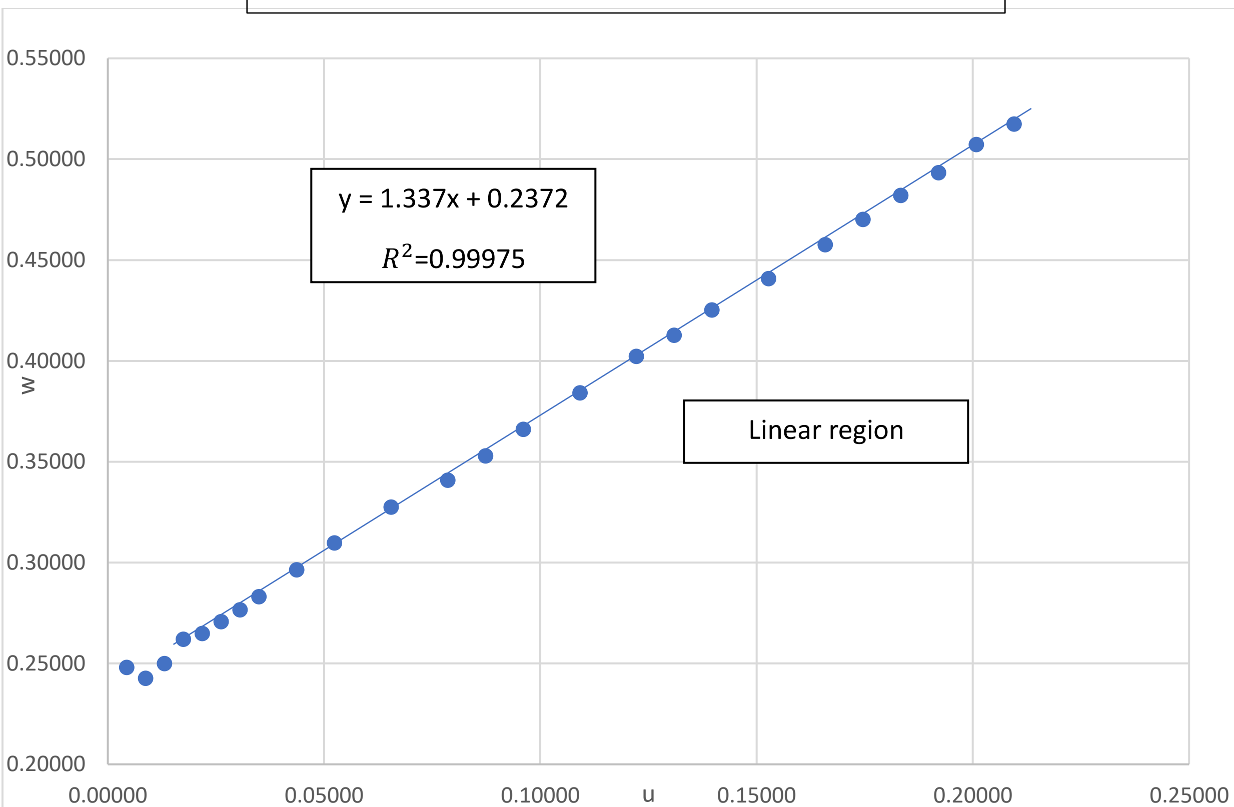
N =1.332 ± 0.002

D.1			D.3	
number	m	θ_m (degree)	u	w
1	1	13.25	0.00436	0.24795
2	2	19.00	0.00873	0.24265
3	3	23.00	0.01309	0.24981
4	4	26.00	0.01746	0.26200
5	5	29.00	0.02182	0.26474
6	6	31.50	0.02619	0.27069
7	7	33.75	0.03055	0.27653
8	8	35.75	0.03492	0.28307
9	10	39.25	0.04365	0.29637
10	12	42.25	0.05238	0.30973
11	15	46.25	0.06547	0.32743
12	18	50.00	0.07857	0.34076
13	20	52.00	0.08730	0.35289
14	22	53.75	0.09603	0.36608
15	25	56.25	0.10912	0.38415
16	28	58.50	0.12222	0.40213
17	30	60.00	0.13095	0.41261
18	32	61.25	0.13968	0.42517
19	35	63.25	0.15277	0.44072
20	38	65.00	0.16587	0.45761
21	40	66.00	0.17460	0.47008
22	42	67.00	0.18333	0.48193
23	44	68.00	0.19205	0.49320
24	46	68.75	0.20078	0.50715
25	48	69.75	0.20951	0.51739

D.2

$$u = \frac{m\lambda}{h} \qquad w = \frac{\frac{m\lambda}{h}\left(n + \frac{m\lambda}{2h}\right)}{1 - \cos\theta_m}$$

D.4



By removing the first 3 points

D.5
B = 1.337
A = 0.2372

D.6
$N_B = 1.337 \pm 0.005$
$N_A = 1.3319 \pm 0.0005$

Theoretical calculations:

B.2 & C.2:

$$m = \frac{H}{\lambda}\left(\sqrt{n^2 - N^2 sin^2\theta_m} - Ncos\theta_m\right) - \frac{H}{\lambda}(n - N)$$

$$\theta_m \ll 1: \qquad \sin\theta_m \approx \theta_m \quad ; \qquad \cos\theta_m \approx 1 - \frac{\theta_m^2}{2}$$

$$m = \frac{H}{\lambda}\left(n\sqrt{1 - \frac{N^2\theta_m^2}{n^2}} - N\left(1 - \frac{\theta_m^2}{2}\right)\right) - \frac{H}{\lambda}(n - N)$$

$$m = \frac{H}{\lambda}\left(n\left(1 - \frac{N^2\theta_m^2}{2n^2}\right) - N\left(1 - \frac{\theta_m^2}{2}\right)\right) - \frac{H}{\lambda}(n - N)$$

$$m = \frac{H}{2\lambda}N\left(1 - \frac{N}{n}\right)\theta_m^2$$

$$N = 1: \; m = \frac{H}{2\lambda}\left(1 - \frac{1}{n}\right)\theta_m^2$$

D.2:

$$m = \frac{h}{\lambda}\left(\sqrt{n^2 - N^2 sin^2\theta_m} - Ncos\theta_m\right) - \frac{h}{\lambda}(n - N)$$

$$\frac{m\lambda}{h} + n - N(1 - cos\theta_m) = \sqrt{n^2 - N^2 sin^2\theta_m}$$

$$\left(\frac{m\lambda}{h}\right)^2 + 2n(\frac{m\lambda}{h}) + n^2 + N^2(1 - cos\theta_m)^2 - 2N(1 - cos\theta_m)(\frac{m\lambda}{h} + n) = n^2 - N^2 sin^2\theta_m$$

$$\left(\frac{m\lambda}{h}\right)^2 + 2n\left(\frac{m\lambda}{h}\right) + N^2(2 - 2cos\theta_m) - 2Nn(1 - cos\theta_m) - 2N(1 - cos\theta_m)(\frac{m\lambda}{h}) = 0$$

$$\frac{\frac{m\lambda}{h}\left(n + \frac{m\lambda}{2h}\right)}{1 - \cos\theta_m} + N^2 - Nn - N\left(\frac{m\lambda}{h}\right) = 0$$

$$\frac{\frac{m\lambda}{h}\left(n + \frac{m\lambda}{2h}\right)}{1 - \cos\theta_m} = N(n - N) + (\frac{m\lambda}{h})N$$

$$w = N(n - N) + uN$$

Error calculations:

Linear equation slope and intercept uncertainties:

$$\Delta B = B\sqrt{\frac{1}{n-2}\left(\frac{1}{r^2}-1\right)} \qquad ; \qquad \Delta A = \Delta B\sqrt{\bar{x}^2+\sigma_x^2}$$

C.5:

$$B = \frac{H}{2\lambda}\left(N-\frac{N^2}{n}\right) \Rightarrow \left(N-\frac{N^2}{n}\right) = \frac{2\lambda}{H}B \equiv c$$

$$\Rightarrow \frac{\Delta c}{c} = \sqrt{\left(\frac{\Delta B}{B}\right)^2+\left(\frac{\Delta H}{H}\right)^2}$$

$$\left(N-\frac{N^2}{n}\right) = c \Rightarrow N = \frac{n}{2} \pm \sqrt{\left(\frac{n}{2}\right)^2 - c\,n}$$

A negative sign is unacceptable in this equation.

$$N = \frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 - c\,n} \Rightarrow \Delta N = \frac{n}{2\sqrt{\left(\frac{n}{2}\right)^2 - c\,n}}\Delta c$$

D.6:

$$N_A(n-N_A) = A \Rightarrow N_A = \frac{n}{2} \pm \sqrt{\left(\frac{n}{2}\right)^2 - A}$$

A negative sign is unacceptable in this equation.

$$N_A = \frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 - A} \Rightarrow \Delta N_A = \frac{\Delta A}{2\sqrt{\left(\frac{n}{2}\right)^2 - A}}$$

Of course, this is calculated by ignoring the error of u and w. Since there is an h value in u and w, the h error causes an error in this quantity that is not included.

Problem E2 - Marking Scheme

A.1	Correct data points ≥ 25 (0.8 pt) If data points are less than 25, (-0.05 pt) for each missing data points. For each missing or wrong number in third column (-0.02 pt)	0.8 pt

A.2	25 data points are properly plotted (0.3 pt) If data points are less than 25, (-0.02 pt) for each missing data points. Missing trend line (- 0.05 pt) Missing each axis titles (-0.02 pt) Too small (-0.05 pt)	0.3 pt

A.3	Correct values of slope (0.05 pt) and intercept (0.05 pt) according to data.	0.1 pt

A.4	Correct answer (0.7 pt) and uncertainty (0.1 pt) The value of h depends on the setup number. First interval less than 2% difference with setup value (0.7pt) Second interval less than 3% difference with setup value (0.5pt) Third interval less than 4% difference with setup value (0.3pt) Correct value of uncertainty (0.1 pt)	0.8 pt

B.1	Correct data points ≥ 15 (0.6 pt) If data points are less than 15, (-0.05 pt) for each missing data points. For each missing or wrong number in third column (-0.02 pt)	0.6 pt

B.2	Correct method (0.05 pt) and correct final equation (0.05 pt)	0.1 pt

B.3	15 data points are properly plotted (0.2 pt) If data points are less than 15, (-0.02 pt) for each missing data points Missing trend line (- 0.04 pt) Missing each axis titles (-0.02 pt) Too small (-0.04 pt)	0.2 pt

B.4	Correct values of slope (0.05 pt) and intercept (0.05 pt) according to data.	0.1 pt

B.5	Correct answer (0.5 pt) and uncertainty (0.1 pt) The value of h depends on the setup number. First interval less than 3% difference with setup value (0.5pt) Second interval less than 5% difference with setup value (0.3pt) Correct value of uncertainty (0.1 pt)	0.6 pt

C.1	Correct data points ≥ 15 (0.6 pt) If data points are less than 15, (-0.05 pt) for each missing data points. For each missing or wrong number in third column (-0.02 pt)	0.6 pt

C.2	Correct method (0.05 pt) and correct final equation (0.05 pt)	0.1 pt

C.3	15 data points are properly plotted (0.2 pt) If data points are less than 15, (-0.02 pt) for each missing data points Missing trend line (- 0.04 pt) Missing each axis titles (-0.02 pt) Too small (-0.04 pt)	0.2 pt

C.4	Correct values of slope (0.05 pt) and intercept (0.05 pt) according to data	0.1 pt

C.5	Correct answer (0.45 pt) and uncertainty (0.15 pt) First interval $1.31 < N < 1.34$ (0.45 pt) Second interval $1.30 < N < 1.35$ (0.25 pt) Uncertainty formula (0.05 pt) and correct value of uncertainty (0.1 pt)	0.6 pt

D.1	Correct data points ≥ 25 (0.7 pt) If data points are less than 25, (-0.05 pt) for each missing data points.	0.7 pt

D.2	correct final equation for u (0.3 pt) and w (0.5 pt)	0.8 pt

D.3	Correct u, w ≥ 25 (1.2 pt) If the number of u, w are less than 25, (-0.05 pt) for each missing or wrong data points.	1.2 pt

D.4	25 data points are properly plotted (0.3 pt) If data points are less than 25, (-0.02 pt) for each missing data points Missing each axis titles (-0.02 pt) Too small (-0.05 pt)	0.3 pt

D.5	Finding the right linear region and drawing trendline (0.05 pt) more than 15 data points in linear region (0.05 pt) Correct values of slope (0.05 pt) and intercept (0.05 pt) according to data.	0.2 pt

D.6	Correct answer (1.3 pt) and uncertainty (0.3 pt) First interval $1.30 < N_B < 1.35$: 0.7 pt Second interval $1.28 < N_B < 1.37$: 0.4 pt First interval $1.30 < N_A < 1.35$: 0.6 pt Second interval $1.27 < N_A < 1.38$: 0.3 pt correct value of uncertainty for N_B (0.1 pt) Uncertainty formula for N_A (0.1 pt) and correct value of uncertainty for N_A (0.1 pt)	1.6 pt