Heat Conduction in a Copper Rod (10 points)

The Experimental Setup

In this experiment a 57.0 cm long copper rod with a diameter of 1.20 cm has been placed across the length of a metal box which is supported by square flanges (Figure 1). The metal box serves to isolate the airflow inside the box.

Figure 1: General view of the experimental equipment

As shown in Figure 2, 7.0 cm from eachside of the two ends of the rod is insulated by styrofoam. Seven holes have been drilled in the rod at equal distances of 7.0 cm; each hole has a depth of 0.6 cm and a temperature sensor (a thermistor) has been placed inside each drilled hole. These sensors have been numbered 1 through 7, from left to right. A similar sensor, numbered 8, has been placed within the box to monitor the ambient temperature of the box (θ_{b}).

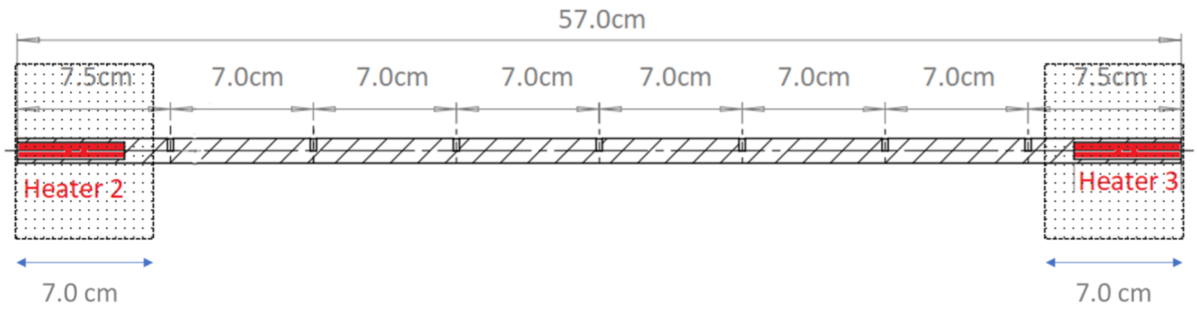

Figure 2: The copper rod and the drilled holes

At the two insulated ends of the rod, two electric heaters with different output powers have been installed inside 5.2 cm-long holes, drilled longitudinally. The left and right heaters are designated as Heaters nos. 2 and 3, respectively. Four fans at the top of the metal box, control the air flow within the box and around the rod. These fans, as well as the copper rod, can be seen from below as shown in Figure 3.

Figure 3: View from below

A control panel, which we call AVA-T403, has been installed on the front. All the switches for turning on the various components of the experiment: the fans, the heaters, the timer, etc. are on this panel and all the measurements made by the sensors are displayed on its monitor. The power necessary to run all the instruments is provided by a 24 V power supply when connected to the power outlet. It lies next to the box.

To the right of the control panel, a smaller 12.0 cm copper rod is installed; it is also covered with styrofoam. Under the styrofoam, Heater no. 1 with an output power of $1.95 \pm 0.06\ \mathrm{W}$ has been inserted inside a longitudinal hole at the right as shown in Figure 4. At the other end, another longitudinal hole has been drilled in which an PT100 (Platinum Thermometer 100) thermistor bundled together with another thermistor (R_9) (similar to the other thermistors) have been inserted. These two sensors and the heater are connected to AVA, and AVA indicates the values of their resistance (in Ω).

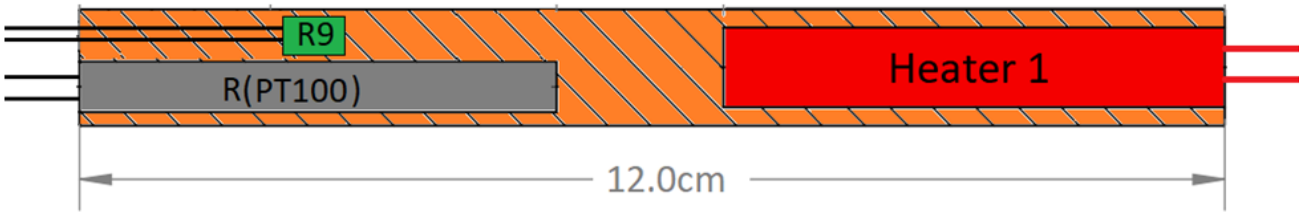

Figure 4: The smaller copper rod

Please observe the following points:

1. Do not touch and activate the buttons on the equipment before it is asked you to do so.

2. You do not need to do anything regarding the setup. Take care not to disturb the setup and not to disconnect any wires.

3. Don't move the setup during the experiment.

4. If the message "Turn off Heater1" appears on the monitor, immediately turn off Heater no. 1.
5. Turning on the heaters will increase the tempereture, so it will take extra time for the system to reach its steady state, make sure that you do not turn on a heater unnecessarily.
6. Errors need to be calculated and reported whenever the $\pm$ sign is present in the answer sheet.
7. Regression, (denoted by reg in the formula below and shown as r on the calculator), is a number between 1 and -1 showing how much the data can be fitted to a line. If $|\mathrm{reg}| = 1$ it means that the data are completely on a line.

We can use the following formula to calculate the uncertainty of the slope b :

$$\Delta b = b\sqrt{\frac{1}{(n-2)}\left(\frac{1}{\mathrm{reg}^2}-1\right)}$$

in which *n* is the number of data points.

The Equipment

1. Generally, Electrical resistors have different behaviors in response to a change in temperature. One of the widely used resistors is PT100 which has a linear behavior over a considerable range of temperatures, i.e.

$$R = R_0(1 + \alpha\theta) \quad (1)$$

in which R_0 is the value of the resistance at the $0\,^{\circ}\mathrm{C}$, α is a constant coefficient (within the range of temperatures for this problem), and θ is the temperature in degrees Celsius. The value of α for the resistor used in this problem is $0.0039083\,^{\circ}\mathrm{C}^{-1}$. For PT100, $R_0 = 100.00\ \Omega$.

2. The thermistors 1 through 9 have a nonlinear behavior in response to changes in temperature, and are usually used for measuring small changes in temperature. The resistance of these thermistors changes with temperature as follows:

$$R' = R'_0 e^{\frac{E_\mathrm{g}}{2k_\mathrm{B}T}} \quad (2)$$

Where $R_0{}'$ is a constant, $k_\mathrm{B} = 8.61733 \times 10^{-5}\,\mathrm{eV}/\mathrm{K}$ is the Boltzmann constant, T is the temperature in kelvins and E_g is the energy gap for the thermistor's semiconductor material. Remember that $T = (\theta + 273.15)\ \mathrm{K}$

3. The digital device AVA was designed by Iranian engineers specifically for this experiment. AVA measures the instantaneous resistances of Thermistors 1 through 7, PT100, and θ_b or R_9. every two seconds. Then, based on the formula 2, reports the temperature of these sensors. However, for Sensor no. 9, only the value of the resistance is displayed. On the left of AVA's monitor, the temperatures of sensors 1 through 7 are displayed in a column. The last row, however, only shows at each instance, either the temperature of sensor 8, or the resistance of Sensor 9 (R_9). You can toggle between these two values by pressing the $\theta_\mathrm{b} - R_9$ button shown in Figure 5.

AVA also has a timer, which works very much like any commercial timer: by pressing the Start/Stop button, the timer starts measuring the time elapsed, and pressing the Start/Stop button again stops the timer. While the timer is working, pressing the Lap button will result in all the displayable values being saved. Pressing the same button when the timer is not working resets the timer, however, the saved data will not be erased. To erase the data the Lap/Reset button has to pressed and held for 5 seconds.

The saved data can be seen by repeatedly pressing the Next/Prev button: pressing Prev shows older data, pressing Next shows newer data. All saved data will be erased in case the device is switched off and on. The device will not turn off automatically.

Figure 5: Start/Stop and Lap buttons. (Further details of keys' functions come as point 3, on the page 4.)

4. As shown in Figure 5, there is one switch for turning the fans on and off ( ) and there are also three switches for turning on the heater ( ). The icon for each switch is stamped below it. The first heater is a 1.95 W heater, the power of second heater is written on the device as shown in Figure 6, and the power of third heater is unknown.

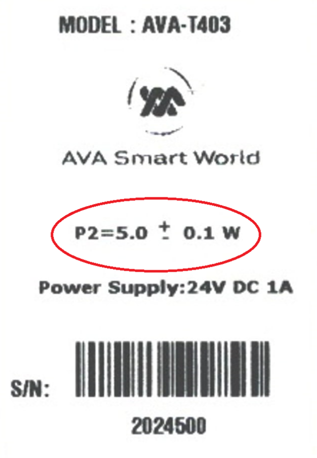

Figure 6: The power of second heater

Theory

In this problem, heat is transferred inside the rod through conduction, and transferred from the rod to the surrounding air through natural or forced convection. Also, due to the heat capacity of the rod, some of the heat injected into the rod is used up to raise the temperature of the rod.

(a) Heat conduction: for a heat conductor in the shape of a rod with no heat loss from its lateral surface, the rate of heat transfer, ${}^{dQ}/_{dt}$, through a differential element (Figure 7) at the steady state is as follows

$$\frac{dQ}{dt} = -kA\frac{d\theta}{dx} \qquad (3)$$

where $d\theta$ is the temperature difference between the two edges of the differential element, A is the cross-sectional area, dx is the length of the differential element, and k is the heat transfer coefficient (heat conductivity) which depends on the type of material the rod is made of.

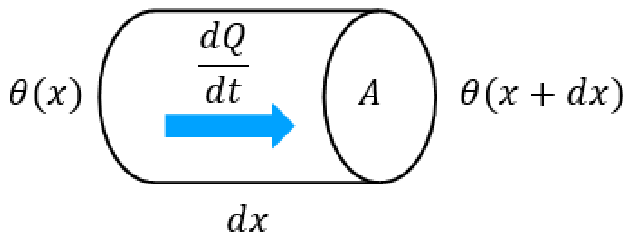

Figure 7: The differential heat conductor element. (heat conduction, in the absence of convection)

(b) Convection: For any object exchanging heat with the air through its lateral surface, the following relationship holds:

$$\frac{dQ}{dt} = -hS\Delta\theta \qquad (4)$$

in which S is the area of the lateral surface, $\Delta\theta$ is the temperature difference between the object and the surrounding air, and h is the convective heat transfer coefficient which is a function of the shape of the object and the nature of the heat flow through the sides of the object.

The Experiment:

In order to save time, we recommend that you turn on Heater 2 and the fans which are needed in Part B. Make sure that the other heaters are not turned on.

Part A: The short copper rod (3.9 points)

A-0	Write numbers 0 to 9 in the table.	0 pt

Before turning on Heater 1, the small rod is at the same temperature as its environment. Tasks A-1 to A-3 are related to the heating process and tasks A-4 to A-7 correspond to the cooling process of the rod.

A-1	Record the initial value of resistance R_{env} (resistance of PT100) when it is at the temperature of its environment. Using Eq.1, find this temperature.	0.2 pt

Let us denote the total heat capacity of the rod and the heater and the sensors by C_{S}. To find C_{S} we should turn on Heater 1 and measure the change in the value of resistance for at least 150 seconds. Note that there is a time delay in the heating and cooling of the sensors.

A-2	In time intervals of approximately 10 seconds record the value of R. Do this at least 15 times.	0.5 pt

A-3	Draw a diagram for Part A-2, fit a line to your data and find its slope. Using the slope find C_S.	0.8 pt

Wait until the value of R reaches $120\ \Omega$ and then turn off Heater 1. The temperature of the rod and the resistance R will start to decrease slowly after a few seconds. When PT100 is cooling down, the resistance is given by:

$$R - R_\mathrm{env} = Ae^{-\gamma t} \qquad (5)$$

in which A and γ are constants.

A-4	At several different instances of time, measure the value of $R - R_\mathrm{env}$.	0.5 pt

A-5	Make a semi-logarithmic plot of your data in part **A-4**. Then find γ.	0.7 pt

The insulator around the rod causes the resistors to reach thermal equilibrium sooner, and, the rod to have a more uniform temperature profile.

A-6	Measure and record the resistance R_9 of Thermistor 9 in terms of R. Measure at least seven different values of R, preferably in the $114 - 120\ \Omega$ range.	0.5 pt

A-7	Draw a diagram of the resistance R_9 versus $1/T$ on the given semi-logarithmic graph and find the magnitude of the energy gap E_g in units of **eV**.	0.7 pt

Part B: The long copper rod (4.1 points)

Wait for the rod to reach a steady state i.e. the measured temperatures at all points remain constant, and then answer the following questions. We shall denote the temperatures of Thermistors 1-7 by θ_1 through θ_7 respectively. The location of Thermistor 1 corresponds to $x = 0$.

B-1	When the rod reaches steady state, measure and record the temperature θ_b and θ_x for the seven points on the rod.	0.4 pt

In order to save time, take a look at Part C and then continue Part B.

B-2	On semi-log paper, draw a diagram for the difference between the temperature of the box and the temperature at point x, $(\theta_x - \theta_\mathrm{b})$ along the length of the rod.	0.4 pt

It can be shown that as a function of the distance from Heater 2 along the length of the line, the temperature obeys the following relation:

$$\theta_\mathrm{x} = \theta_\mathrm{b} + Ae^{-\lambda x} + Be^{\lambda x} \qquad (6)$$

in which θ_b the ambient temperature of the box, A and B are constants, and $\lambda = \sqrt{\frac{2h}{kr}}$ where r is the radius of the rod. As a first step, we can ignore the data corresponding to large values of x and take B to be zero. In this case we can find λ and A to a first approximation, let us call them $\lambda^{(0)}$ and $A^{(0)}$:

B-3	Use the temperatures θ_1 through θ_5 and find $\lambda^{(0)}$ and $A^{(0)}$ using the diagram of Part **B-2**.	0.6 pt

The temperature at the end of the rod farthest from the heater does not change with x. Assume this happens around a distance $x = d$. One can use this to determine B in terms of λ, A, and d:

B-4	Express B in terms of λ, A, and d, and for $d = 44.0$ cm. Find its numerical value using the results of Part **B-3**. Denote this quantity as $B^{(1)}$.	0.4 pt

Now we can use the value obtained for B to correct the previous calculation. To do so, assume:

$$\theta_x{}' = \theta_x - B^{(1)}e^{\lambda^{(0)}x}$$

B-5	Find $\theta_1{}'$ through $\theta_7{}'$ and complete the columns added to table B-1.	0.4 pt

B-6	Draw a new diagram to obtain the values for λ and A Denote them by $\lambda^{(1)}$ and $A^{(1)}$ respectively.	1.0 pt

To obtain accurate approximations, the corrections should be repeated many times, but in the end, we'll find that the final answer is close to $\lambda = \frac{\lambda^{(0)}+\lambda^{(1)}}{2}$ and $A = \frac{A^{(0)}+A^{(1)}}{2}$.

B-7	By balancing the input and output powers of the copper rod, find h and k.	0.9 pt

Part C: Measuring the unknown power (2.0 points)

While the fans are on, turn on Heaters 2 and 3, and wait until the temperature reaches equilibrium at all points of the copper rod. (This will take about 15 minutes.)

C-1	Measure and record the temperatures θ_1 through θ_7.	0.4 pt

It can be shown that, in this case, the temperature in terms of x varies as follows

$$\theta_x - \theta_\mathrm{b} = A' \cosh(\lambda(x - x_0))$$

in which A' is a constant and $\cos h(u)$ is the hyperbolic cosine function of u defined as:

$$\cos h\left(u\right) = \frac{e^u + e^{-u}}{2}$$

C-2	Draw the diagram of temperature versus distance and find x_0.	0.6 pt

C-3	Using your own method to find the effective power of Heater 3. Explain your method as clearly as possible by writing down explicitly the mathematical formulas you have used to arrive at your results.	1.0 pt

A.0 (0pt)

0	1	2	3	4	5	6	7	8	9

A.1 (0.2 pt)

R_{en}= ±

θ_{en} = ±

A.2 (0.5 pt)

n	$R(\Omega)$	$t(s)$
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

A.3 (0.8 pt)

$\Delta R/\Delta t=$

$\Delta\theta/\Delta t=$

$C_s =$ $\pm$

A.4 (0.5pt)

n	$R(\Omega)$	$t(s)$	$(R - R_{en})(\Omega)$
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
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15			
16			
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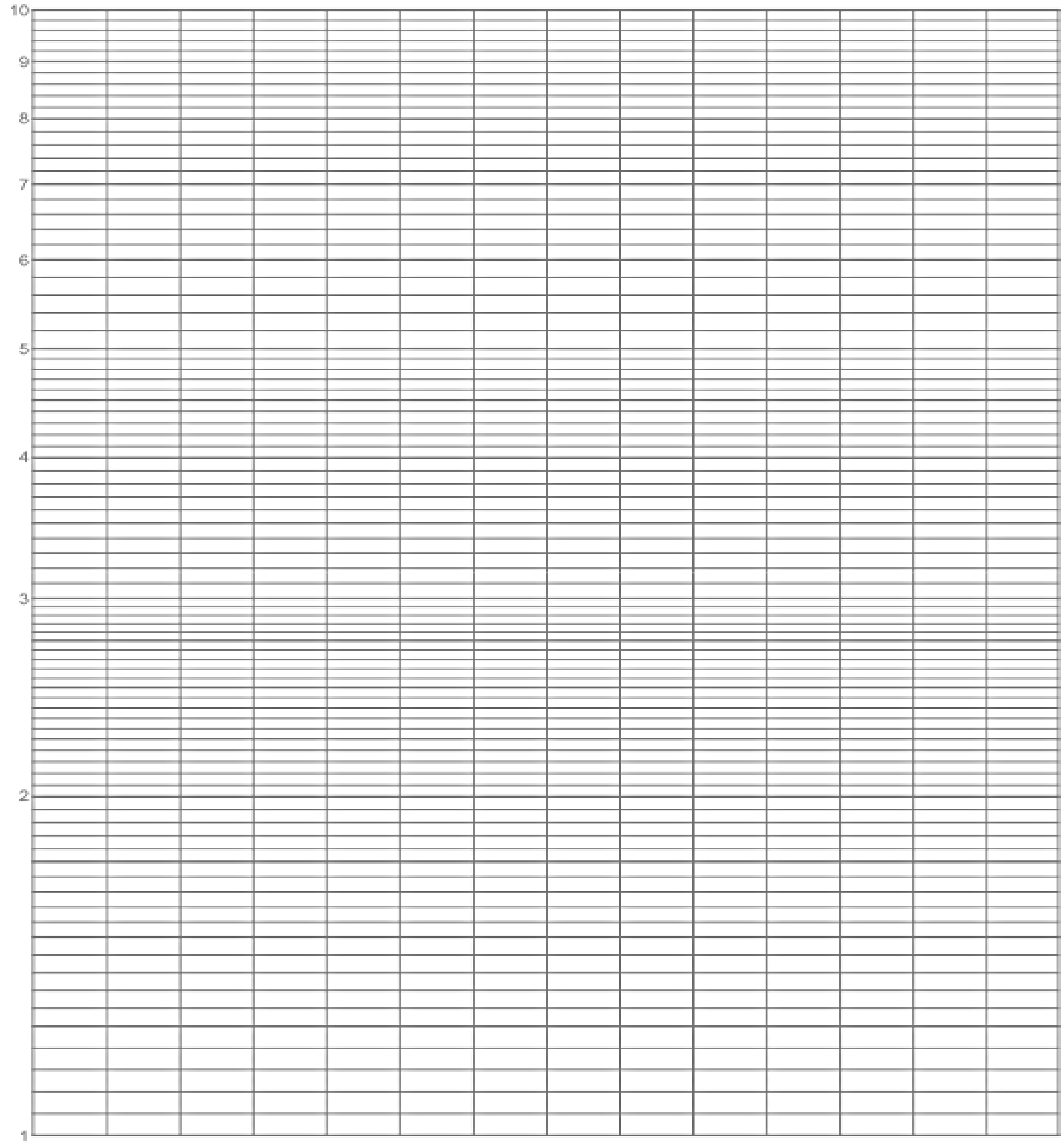
20			

A.5 (0.7 pt)

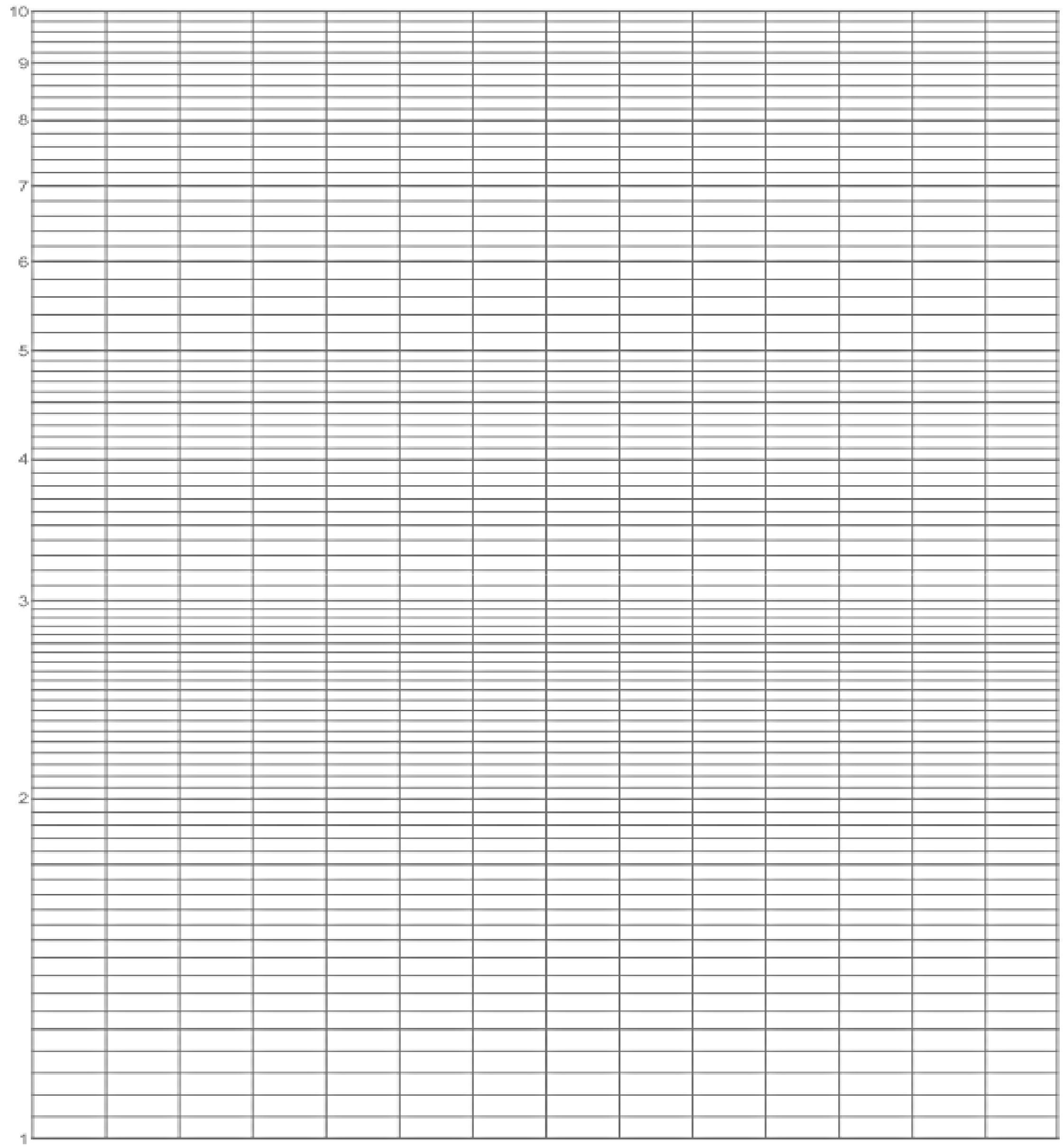


γ= ±

A.6 (0.5 pt)

N	$R_9(\Omega)$	$R(\Omega)$	$T(k)$	$\frac{1}{T}\left(\frac{1}{k}\right)$
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				

A.7 (0.7 pt)



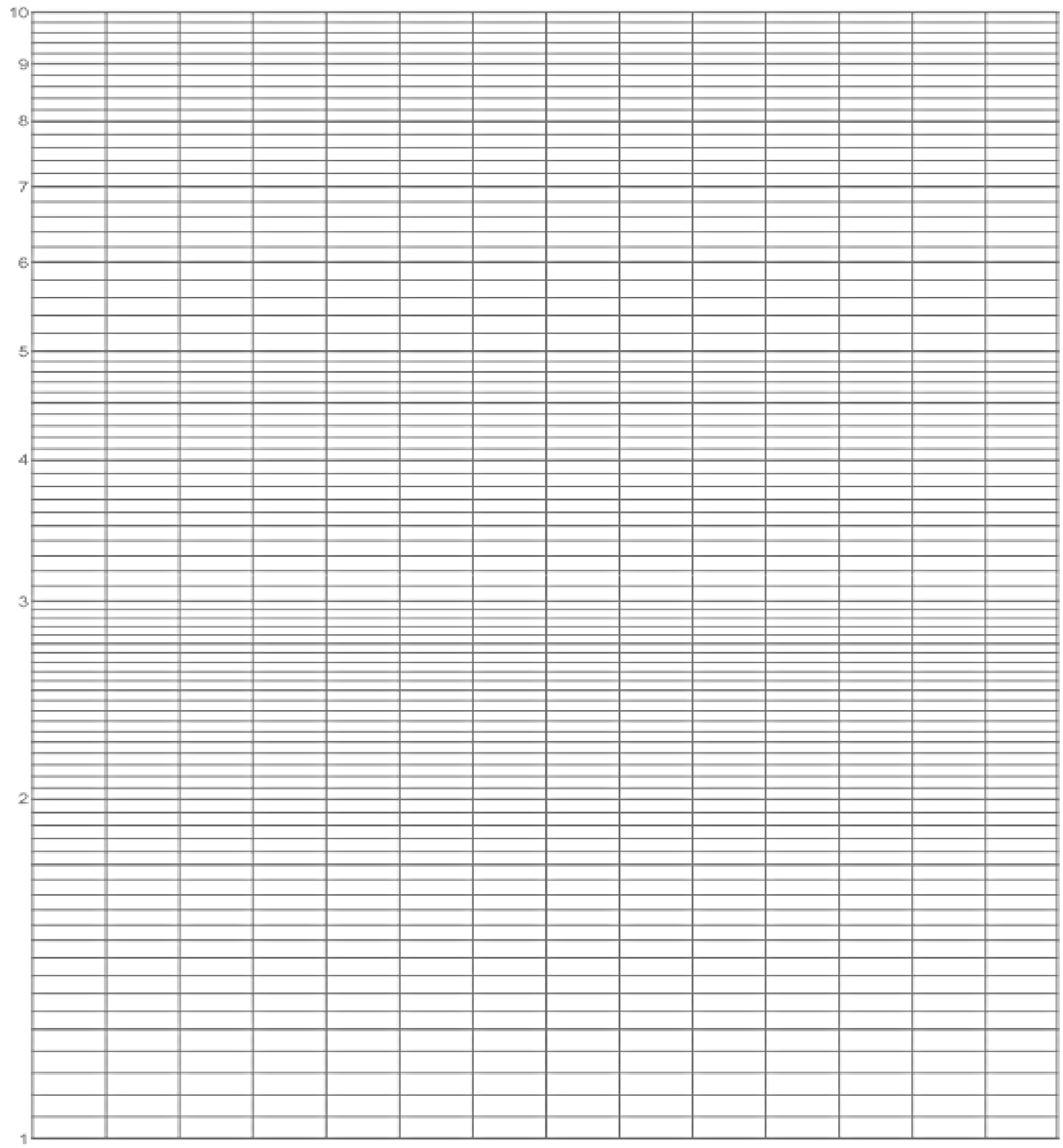
E_g= ±

B

$\theta_b =$

B.1 (0.4pt)				B.5 (0.4pt)	
n	$x(cm)$	$\theta_x(°C)$	$\theta_x - \theta_b(°C)$	$B^{(1)}e^{\lambda^{(0)}x}(°C)$	$\theta'_x - \theta_b(°C)$
1	0				
2	7				
3	14				
4	21				
5	28				
6	35				
7	42				

B.2 (0.4 pt)



B.3 (0.6 pt)

$A^{(0)} =$

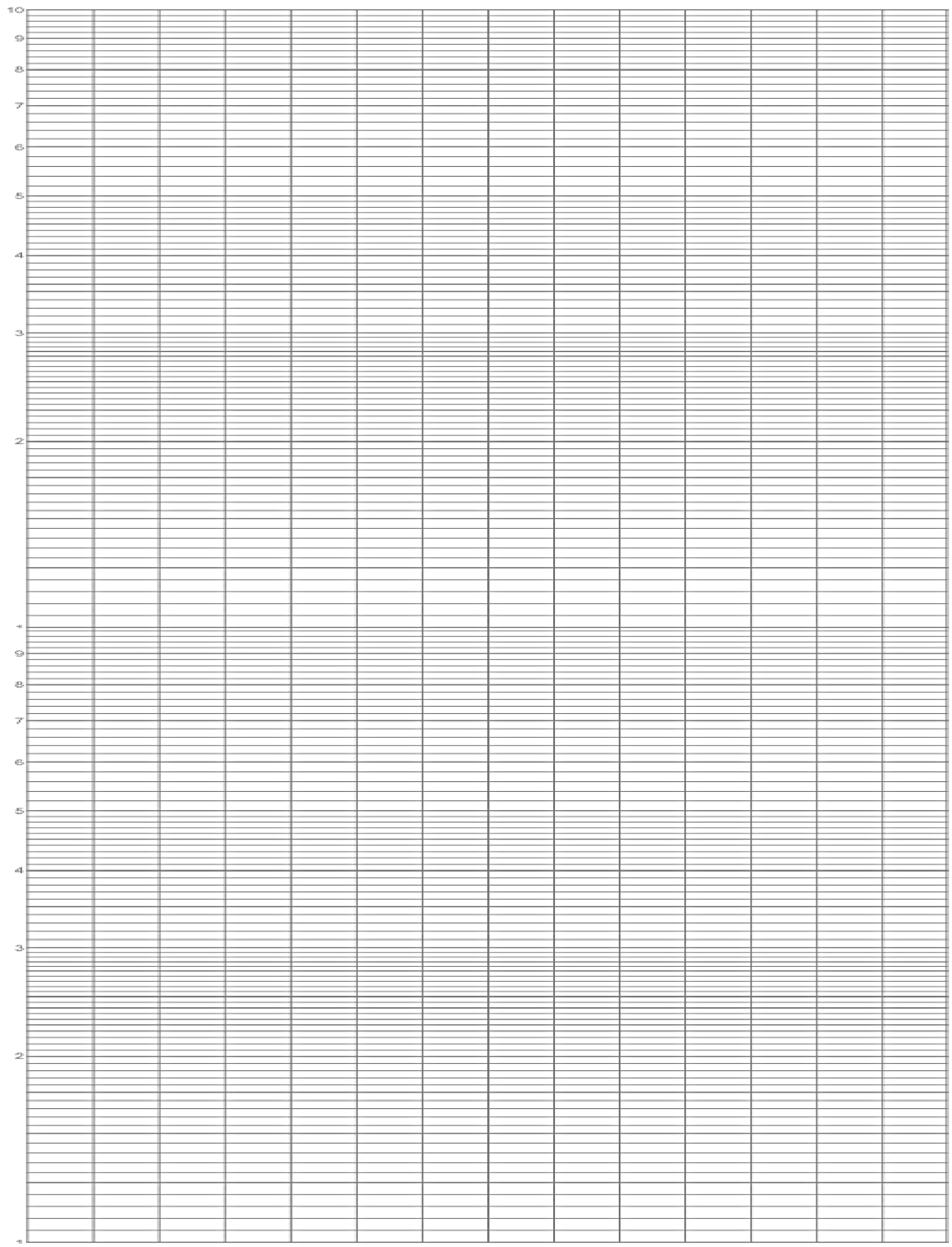
$\lambda^{(0)} =$ $\pm$ $(\frac{1}{cm})$

B.4 (0.4 pt)

$B =$

$B^{(1)} =$

B.6 (0.4 pt)



B.6

$A^{(1)}$ =

$\lambda^{(1)}$ = ±

B.7 (0.9 pt)

λ = ±

h = ±

k = ±

C.1 (0.4 pt)

$\theta_b =$

n	$x(cm)$	$\theta_x(°C)$	$(\theta_x - \theta_b)(°C)$			
1	0					
2	7					
3	14					
4	21					
5	28					
6	35					
7	42					

C.2 (0.6 pt)

C.3 (1.0 pt)

$P_3 =$ $\pm$

Diffraction from Phase Steps (10 points)

The Equipment Box

figure 1: The equipment box

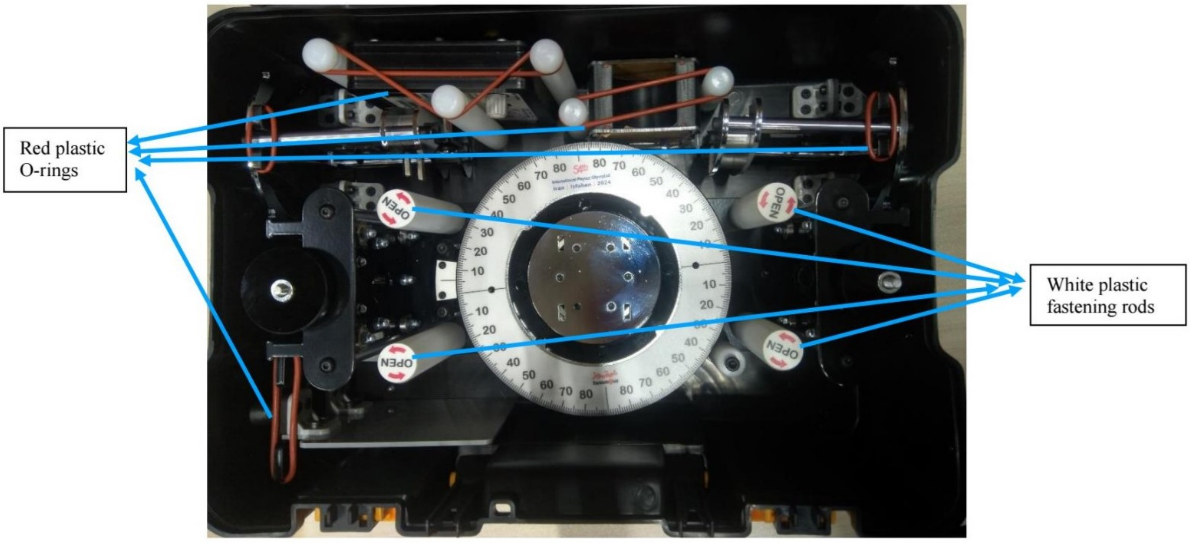


figure 2: Inside the box, upview

To release the setup, you need to unscrew the white plastic fastening rods in the direction indicated on the top. You can pull out the main platform from the box as shown in Figure 3. After this, remove the red plastic O-rings indicated in Figure 2 and remove the other components inside the box one by one. To remove the instruments hold them by their metallic parts or by their outer surface.

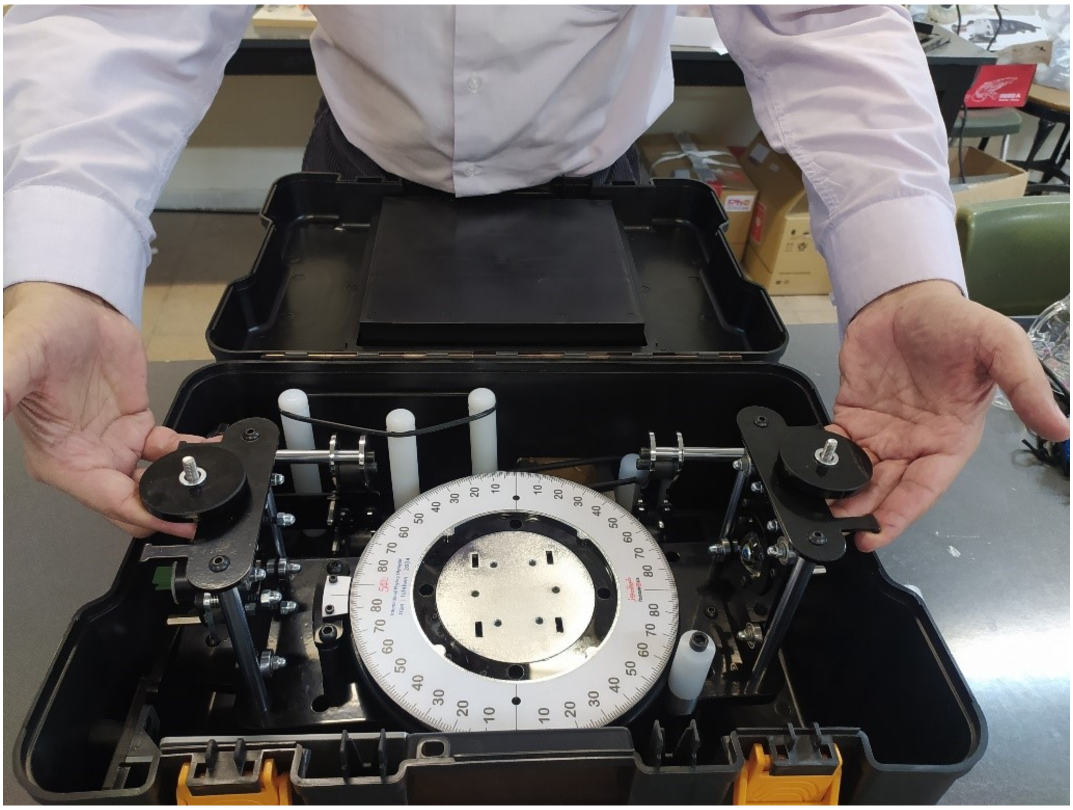


Figure 3: Extracting the main platform from the box.

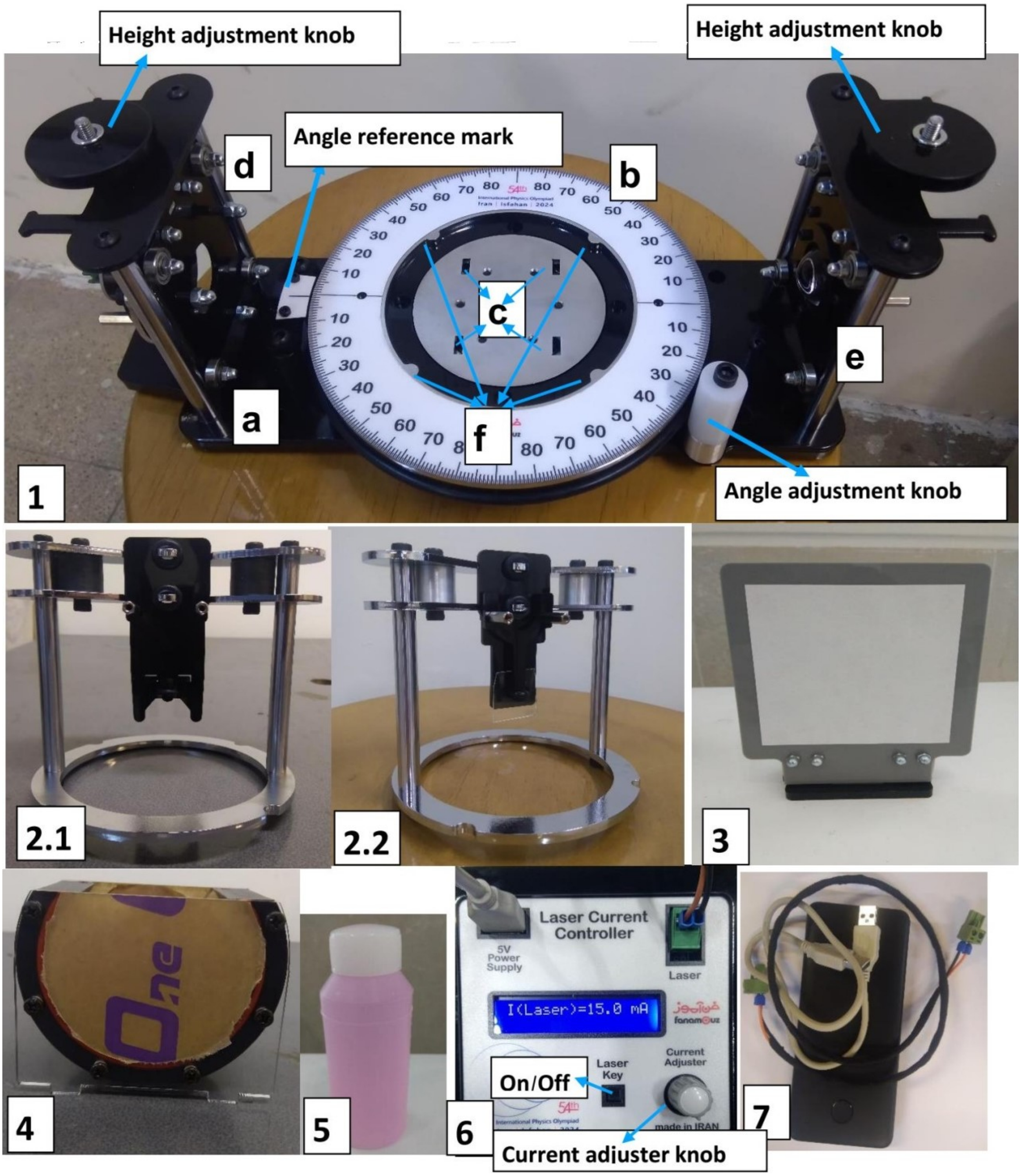


Figure 4: The experimental setup and its components

The setup (Details)

1. **The main platform** of the experimental setup, which consists of:

(a) A horizontal base.

(b) A white rotating protractor: it can be rotated using the white plastic knob next to it. A reference mark on the metallic plate can be used to read the angle (see Figure 4-1).

(c) A circular plate with 4 square holes to hold the container of an unknown liquid.

(d) A red laser and (e) convex lenses for magnifying the diffraction pattern, installed on the walls at the two sides of the platform: their height can be adjusted by turning the knobs at the top.

(f) Four protrusions on the inner wall of the protractor to hold the glass pieces' holders.

2. **Holders S1 (2.1) and S2 (2.2)**: each Holder stands on the circular metallic plate concentric with protractor and the four protrusions (Figure 4-1f) keep it fixed. The S1 holder includes a black piece which holds a thin microscope slide. The lower edge of the slide is completely free and laser light can be shone onto it. The S2 Holder is quite similar to S1, the only difference being that it holds a thick microscope slide.

3. **The observation screen**: it can be placed at any distance from the setup.

4. **The unknown liquid container**: after removing the protective adhesive paper, it can be placed on the square holes in the middle of the protractor (Figure 4-1c). The effect of container walls on the diffraction pattern is negligible.

5. **The pink liquid**, inside the bottle on your desk, has an unknown refractive index.

6. **The laser electronic board**: it can be turned on by connecting the laser to the board (and the board to the power bank). Use the On/Off switch on the board to turn the laser on or off. The intensity of the laser light can be adjusted by turning the current adjuster knob on the electronic board. Set the intensity of the laser to a level at which your eyes are comfortable.

7. **Power bank and electrical cables**.

Please take note of the following:

1. Do not touch the glass lens and the microscope slides at all, because your fingerprints can affect the results of your experiment, and the slides are rather thin and can easily break.

2. Do not drink the unknown liquid.

3. Do not look directly into the laser.

Theory

When a laser beam is shone at the edge of a transparent slide, a phase difference is introduced between the part that travels through the slide and the part that does not. This phase difference results in a diffraction pattern, the lines of which are parallel to the edge of the slide (see Figure 5).

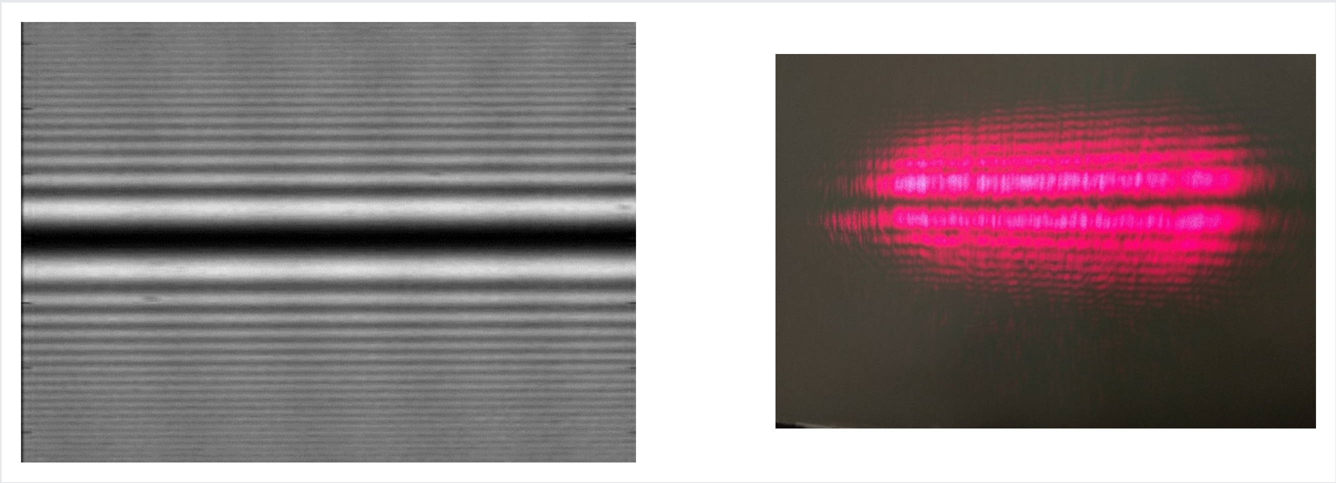


Figure 5: A theoretical diffraction pattern (left), and the diffraction pattern observed in the lab (right).

Let us take the direction of the beam as the z-direction (see Figure 6), and at first, we'll assume that the slide is in the x-y plane and its horizontal edge coincides with the x-axis (i.e. the angle in Figure 6 is equal to zero). In this case the phase difference between the two parts of the beam clearly is:

$$\phi_0 = \frac{2\pi h}{\lambda}(n - N) \qquad (1)$$

where h is the thickness of the slide, λ is the wavelength of the laser beam, N is the refractive index of the environment, and n is the refractive index of the transparent slide.

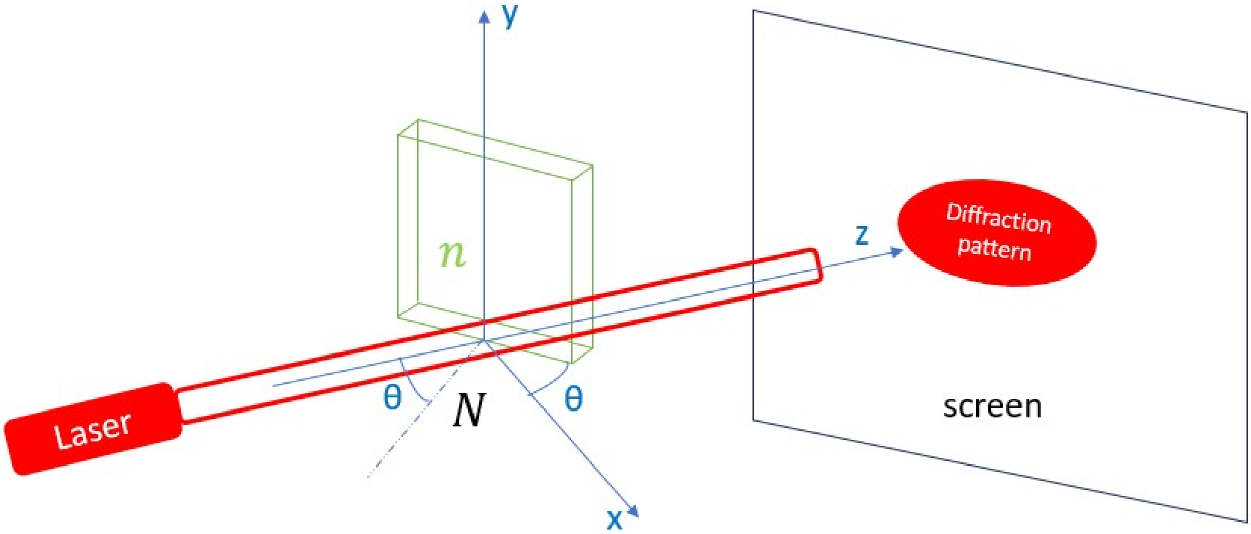


Figure 6: Schematic of the laser beam, the slide, and the screen.

If we rotate the slide around the y-axis so that the normal to the surface of the slide makes an angle of θ with the incident beam, a simple calculation gives the following formula for the phase difference

$$\phi = \frac{2\pi h}{\lambda}\left(\sqrt{n^2 - N^2 \sin^2\theta} - N\cos\theta\right) \qquad (2)$$

Hence the phase difference is a function of θ. If we continuously change this angle, the phase difference increases continuously and the shape of the pattern changes, but when the phase difference reaches 2π, the pattern reverts to its initial shape. We call this full cycle **one fringe shift**. Figure 7 displays the various stages of one fringe shift.

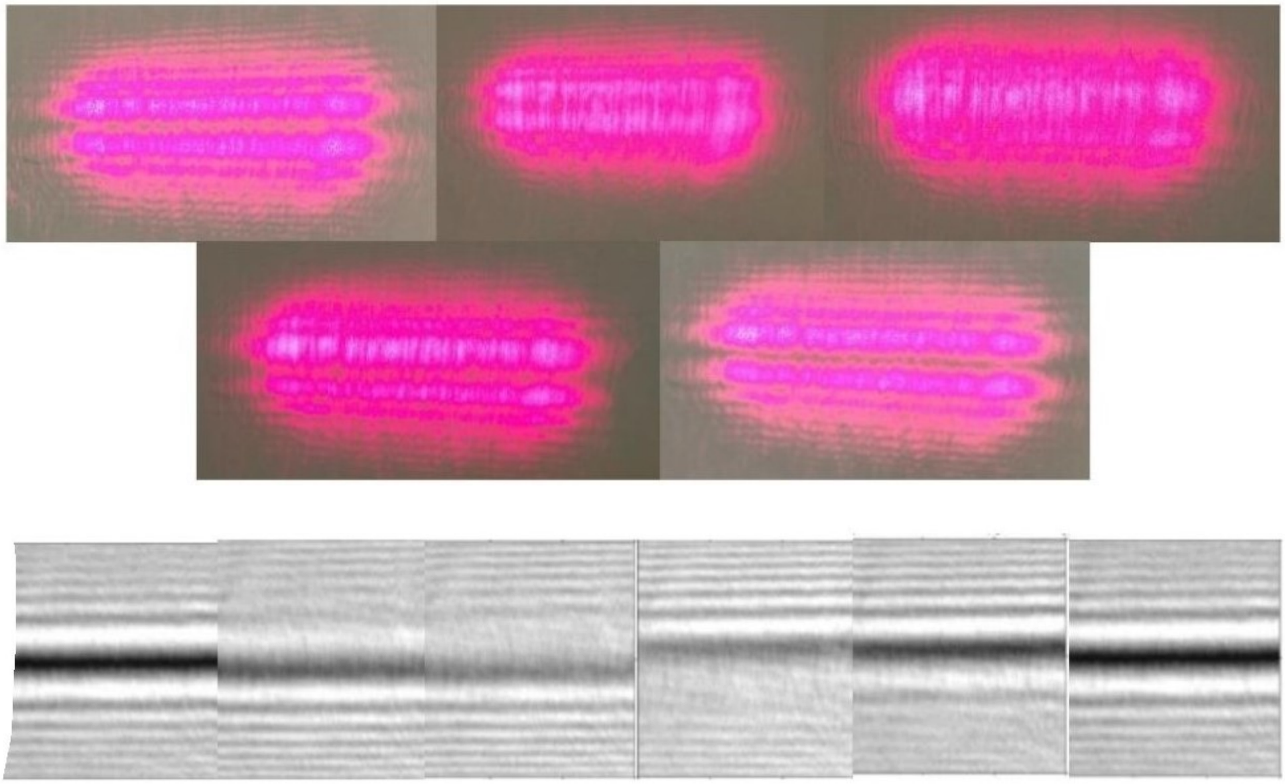


Figure 7: Various stages of fringe shift as seen on the screen in the lab and as predicted theoretically (from left to right, each figure has phase difference equals to φ, φ+4π/9, φ+6π/9, φ+10π/9, φ+14π/9, φ+2π).

We can start from $\theta = 0$ and gradually increase the angle. After m such fringe shifts corresponding to a rotation by $\theta = \theta_\mathrm{m}$, we will have:

$$\phi = \frac{2\pi h}{\lambda}\left(\sqrt{n^2 - N^2\sin^2\theta_\mathrm{m}} - N\cos\theta_\mathrm{m}\right) = 2\pi m + \phi_0 \qquad (3)$$

or:

$$m = \frac{h}{\lambda}\left(\sqrt{n^2 - N^2\sin^2\theta_\mathrm{m}} - N\cos\theta_\mathrm{m}\right) - \frac{\phi_0}{2\pi} \qquad (4)$$

Important note:

1. You only need to calculate the uncertainty in the final results of each part (Errors need to be calculated and reported whenever the $\pm$ sign is present in the answer sheet).
2. You can use the provided calculator to find the slope and the vertical axis intercept of the curves.

3. Note:
   Regression, r, is a number between 1 and -1 showing how much the data can be fitted to a line. If |r|=1 it means data are completely on a line.
   In case we're calculating slope (B) and intercept (A) using a calculator in linear mode, we can use these formulas below in order to calculate their uncertainty:

$$\Delta B = B\sqrt{\frac{1}{(\mathfrak{n}-2)}\left(\frac{1}{r^2}-1\right)}$$

$$\Delta A = \Delta B\sqrt{\overline{x^2}}$$

Which $\mathfrak{n}$ is number of data points we've got, and $\overline{x^2}$ is average of square of X.

- You must calculate uncertainty of the slope and the intercept only by the formulas above.

Part A: Thickness of the thin slide (S1) (2.0 points)

For the following tasks, take the refractive index of the glass components (S1, S2) to be 1.51 and that of air to be 1.00 . Take the wavelength of the red laser to be $650\ \mathrm{nm}$, and ignore any uncertainty in these values.

Turn on the laser. Place the S1 Holder on the protractor, and adjust the height of the laser such that it shines on the bottom edge of the microscope slide. Then adjust the height of the lens until you can observe the diffraction pattern on the screen (this height should almost be equal to the height of the laser beam). Note that the fringes in the diffraction pattern are horizontal. figure 8 shows the experimental setup for part A. Now slowly turn the protractor and observe the fringe shift.

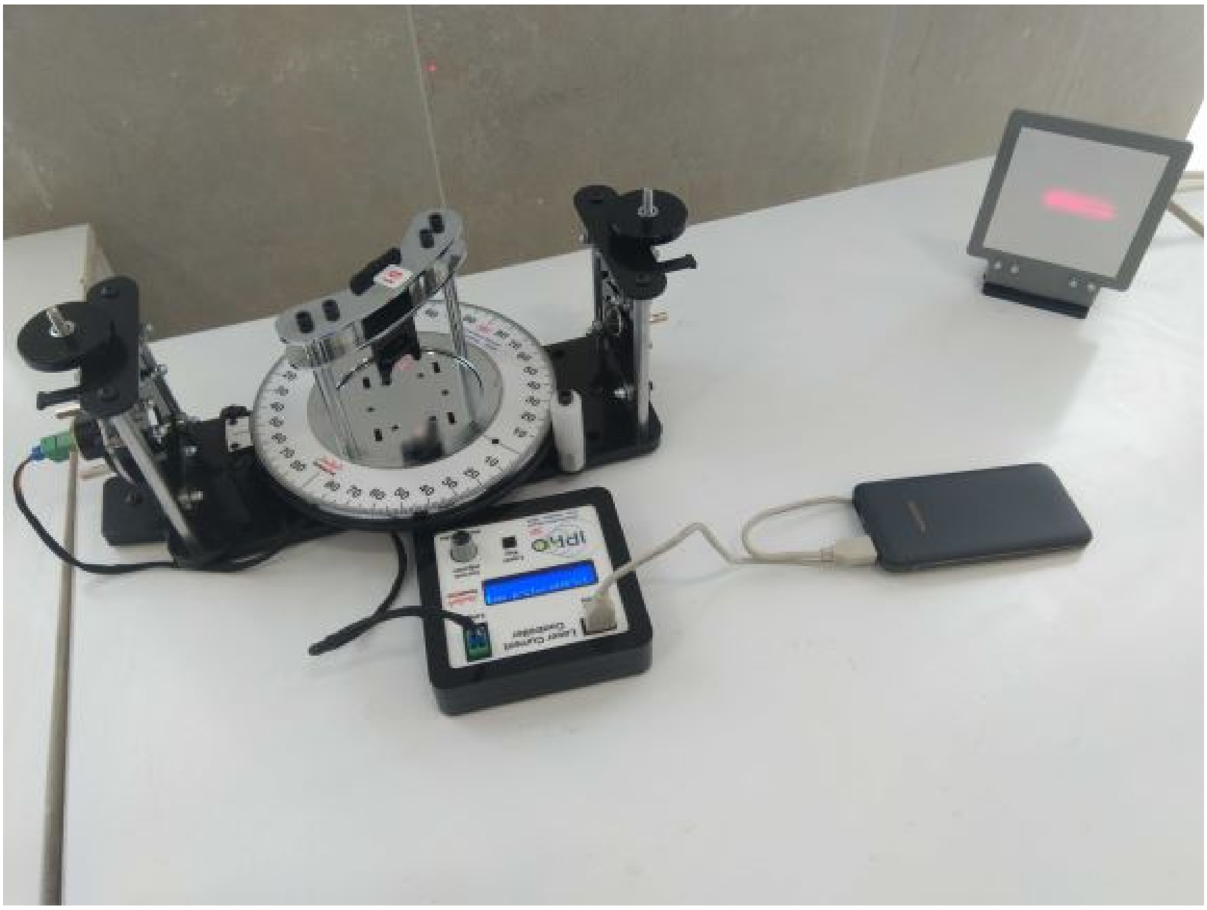


Figure 8: experimental setup in operation (part A)

A-1	Starting with zero degrees, rotate the protractor and go up to 70 degrees. Watch the number of fringe shifts and write down the angle θ_{m} corresponding to each fringe shift number m. Take at least 25 data points and fill out the table.	0.8 pt
A-2	Draw appropriate graph.	0.3 pt
A-3	Find the slope (B) and the vertical axis intercept (A).	0.1 pt
A-4	Using the slope, find the thickness of the thin slide.	0.8 pt

Part B: Thickness of the thick slide (S2) (1.6 points)

Go back to the setup for Part A using the S2 Holder instead of the S1 Holder.

B-1	Repeat the task **A-1** for θ between 0 and 20 degrees and record at least 15 data points.	0.6 pt

B-2	Assuming θ_m in Equation 4 is small enough, expand the relation to the order θ_m^2 and find a linear relation between the fringe shift number and θ_m^2 (assume $N = 1.00$).	0.1 pt

B-3	Draw an appropriate graph.	0.2 pt

B-4	Find the slope and the vertical axis intercept.	0.1 pt

B-5	Using the slope, find the thickness of the thick slide.	0.6 pt

Part C: Finding *N* using the thick microscope slide (S2) (1.6 points)

Pour the unknown liquid into the container. Place the container at the center of the protractor and gently place the S2 Holder back onto the protractor in such a way that the microscope slide is immersed inside the liquid. Adjust the height of the laser and the microscope slide so that the laser beam shines at the boundary between the slide and the ambient liquid. Again, a diffraction pattern will be observed on the screen.

C-1	Repeat the task **B-1** (15 data points up to 20 degrees).	0.6 pt

C-2	Repeat the task **B-2** for arbitrary N.	0.1 pt

C-3	Draw an appropriate graph.	0.2 pt

C-4	Find the slope and the vertical intercept.	0.1 pt

C-5	Find the refractive index of the unknown liquid (N).	0.6 pt

Part D: Finding *N* using the thin microscope slide (S1) (4.8 points)

Put S1 holder instead of S2 inside unknown liquid.

D-1	Repeat the task **A-1** for this case (25 data points up to 70 degrees).	0.7 pt
D-2	Do a simple calculation and eliminate ϕ_0 from Equations 1 and 4 to obtain a relation like $N(n-N)+uN=w$. Find u and w in terms of m, n, h, λ, and θ.	0.8 pt
D-3	Use your calculator to determine u and w.	1.2 pt
D-4	Draw w versus u.	0.3 pt
D-5	In the previous graph, find the linear region and calculate the slope and the vertical axis intercept of the curve.	0.2 pt
D-6	Find the refractive index, first using the slope (N_B), and then using the vertical intercept (N_A).	1.6 pt

A.1			
No.	m	θ_m(deg)	$\sqrt{n^2 - \sin^2\theta_m} - \cos\theta_m$
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			

A.2

A.3

B =

A =

A.4

h = $\pm$

B.1			
No.	m	θ_m (deg)	${\theta_m}^2$ (rad^2)
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			

B.2

B.3

B.4

B =

A =

B.5

H = $\pm$

C.1			
No.	m	θ_m (deg)	${\theta_m}^2$ (rad^2)
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			

C.2

C.3

C.4

B =

A =

C.5

N = $\pm$

D.1			D.3	
No.	m	θ_m (deg)	u	w
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
21				
22				
23				
24				
25				

D.2

u = w =

D.4

D.5

B =

A =

D.6

N_B = ±

N_A = ±