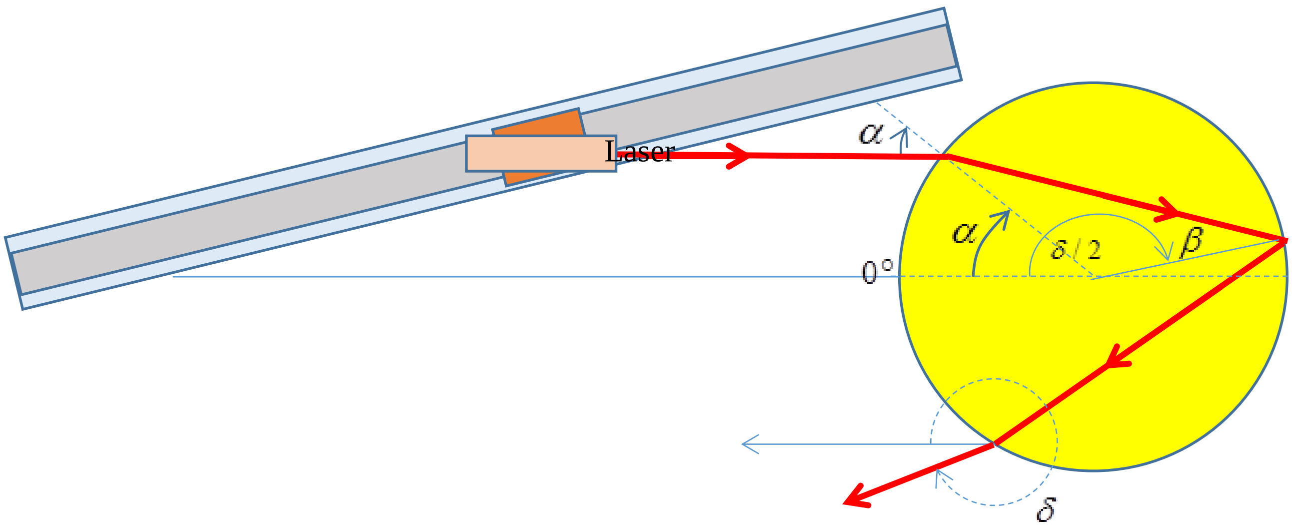
Optical Measurements – Solution

Part A: The refractive index of a disk

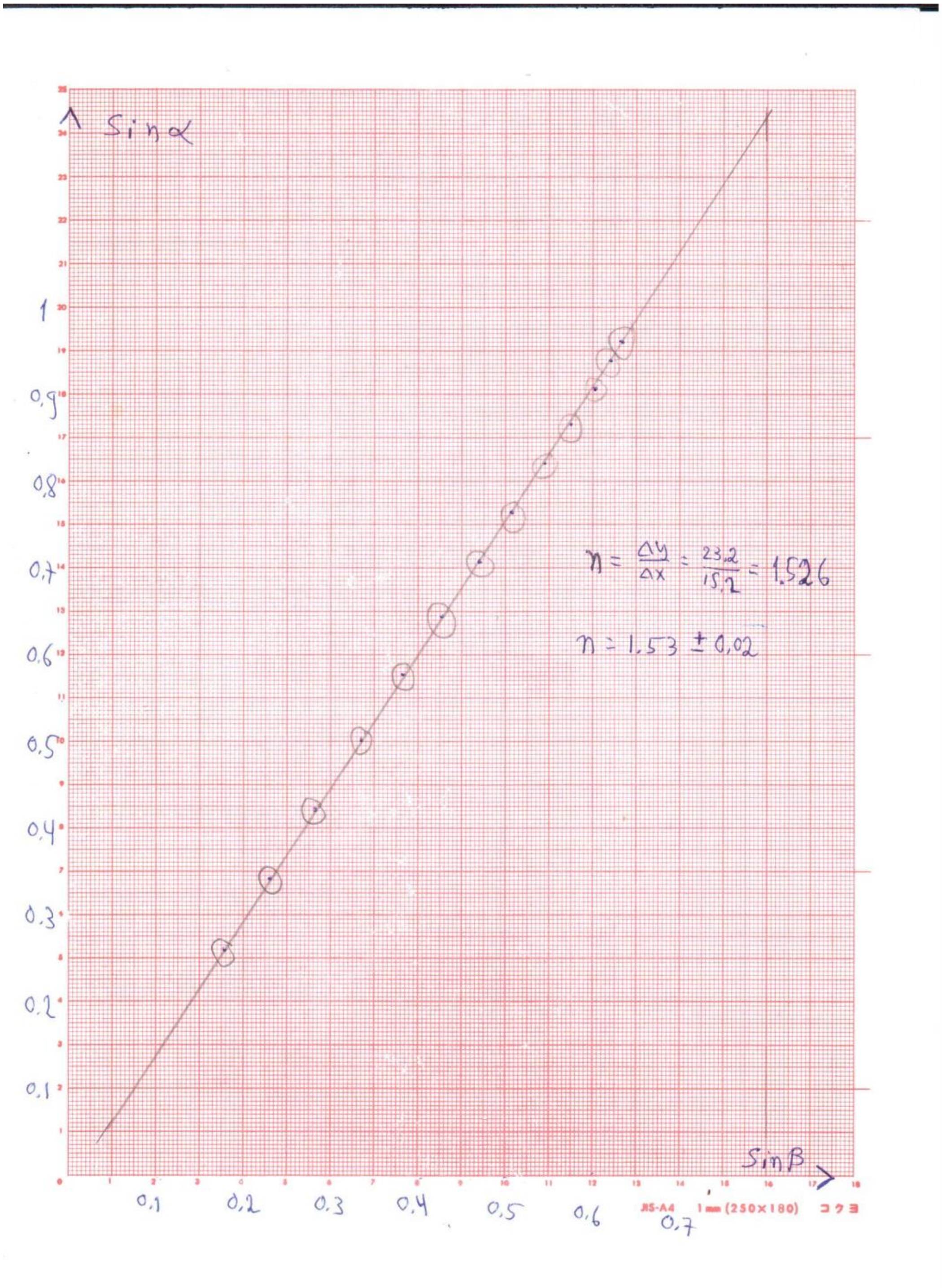
A.1: A sketch of the experimental setup for $N = 3$



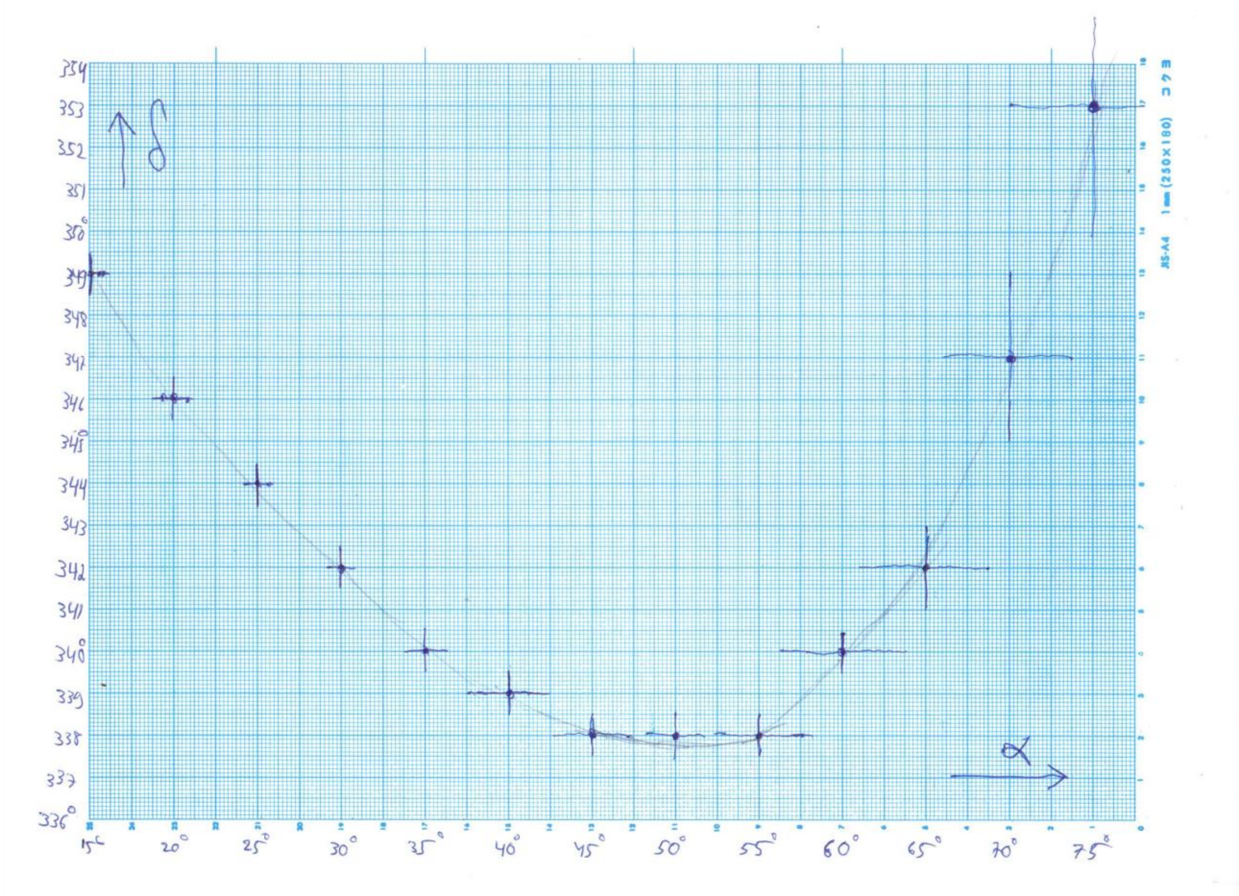
A.1: table of measured and calculated values

$\alpha(\circ)$	$\Delta\alpha(\circ)$	$\delta/2(\circ)$	$\Delta\delta/2(\circ)$	$\delta(\circ)$	$\Delta\delta(\circ)$	$\beta(\circ)$	$\sin\alpha$	$\sin\beta$
15	0.25	174.5	0.25	349	0.5	10.25	0.259	0.178
20	0.25	173	0.25	346	0.5	13.5	0.342	0.233
25	0.25	172	0.25	344	0.5	16.5	0.423	0.284
30	0.25	171	0.25	342	0.5	19.5	0.500	0.334
35	0.5	170	0.25	340	0.5	22.5	0.574	0.383
40	1	169.5	0.25	339	0.5	25.25	0.643	0.427
45	1	169	0.25	338	0.5	28	0.707	0.469
50	1	169	0.25	338	0.5	30.5	0.766	0.508
55	1	169	0.25	338	0.5	33	0.819	0.545
60	1.5	170	0.25	340	0.5	35	0.866	0.574
65	1.5	171	0.5	342	1	37	0.906	0.602
70	1.5	173.5	1	347	2	38.25	0.940	0.619
75	2	176.5	1.5	353	3	39.25	0.966	0.633

A.2:


Sin α
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
n = Δy/Δx = 23,2/15,2 = 1,526
n = 1,53 ± 0,02
Sin β
0,1
0,2
0,3
0,4
0,5
0,6
0,7

A.3:



By observing the remote screen, it is possible to identify the point in which δ is minimal at the highest accuracy.

The values we find are

$\alpha = 49° \pm 0.25°$ and $\delta = 338° \pm 0.5°$

A.4:

When δ is minimal, $\frac{d\delta}{d\alpha} = 0$.

Differentiating the relation $\delta = 2\alpha + (N-1)(180° - 2\beta)$ by α we get:

$2 - 2(N-1)\frac{d\beta}{d\alpha} = 0$ and therefore $\frac{d\beta}{d\alpha} = \frac{1}{N-1}$.

By differentiating Snell's law $\sin\alpha = n\sin\beta$ we get $\cos\alpha = n\cos\beta \cdot \frac{d\beta}{d\alpha} = \frac{n\cos\beta}{N-1}$

Squaring this result, as well as Snell's law and summing the expressions we get:

$$1 = \sin^2\alpha + \cos^2\alpha = n^2\sin^2\beta + \frac{n^2\cos^2\beta}{(N-1)^2}$$

Hence: $\frac{1}{n^2} = \sin^2\beta + \frac{\cos^2\beta}{(N-1)^2}$

We got an explicit relation between the refraction angle β and the refraction index of the material. Due to the multiple reflections inside the disk it is possible, by following all the point in which the beam hits the disk-air interface, to measure the angle β at very high accuracy.

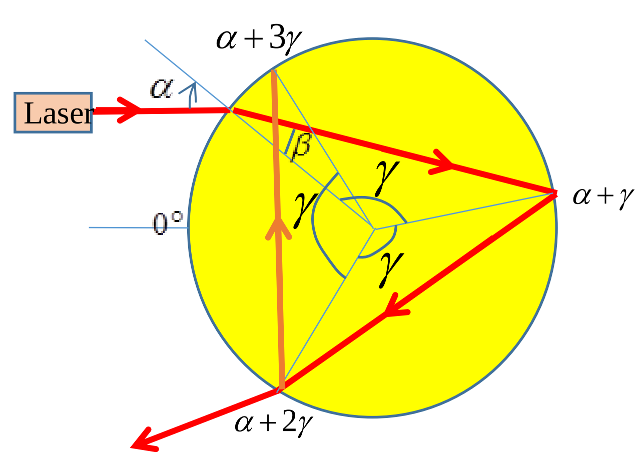
A.5: a sketch showing all the measured quantities:

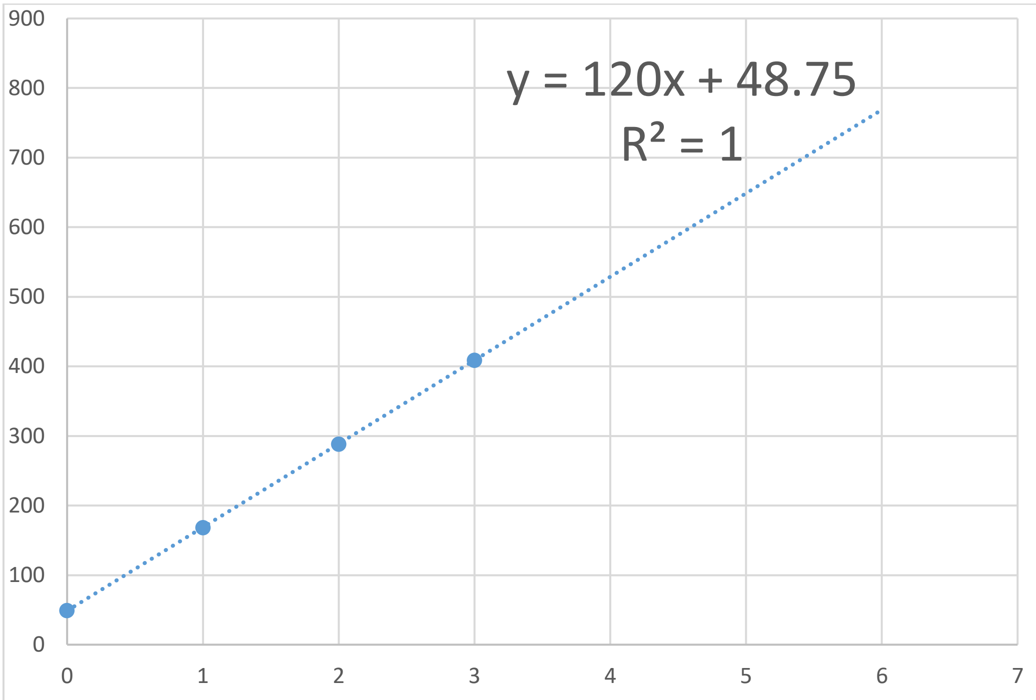
Define the angle $\gamma = 180° - 2\beta$, as shown in the sketch. In fact, after two reflections inside the disk the beam exits at a point very close to the entering point. We will measure the angular location of the points where the beam hits the interface after k reflection, for as many values of k as we can:

k	$\alpha + k\gamma$
0	49
1	168.5
2	288.5
3	409

Note: for the case of $N = 3$ it is not possible to measure for $k > 3$ as in this case, starting from $k = 3$ the impact points co-inside with previous points.

Next we draw a graph of $y = \alpha + k\gamma$ vs. k and find the linear regression slope, γ:



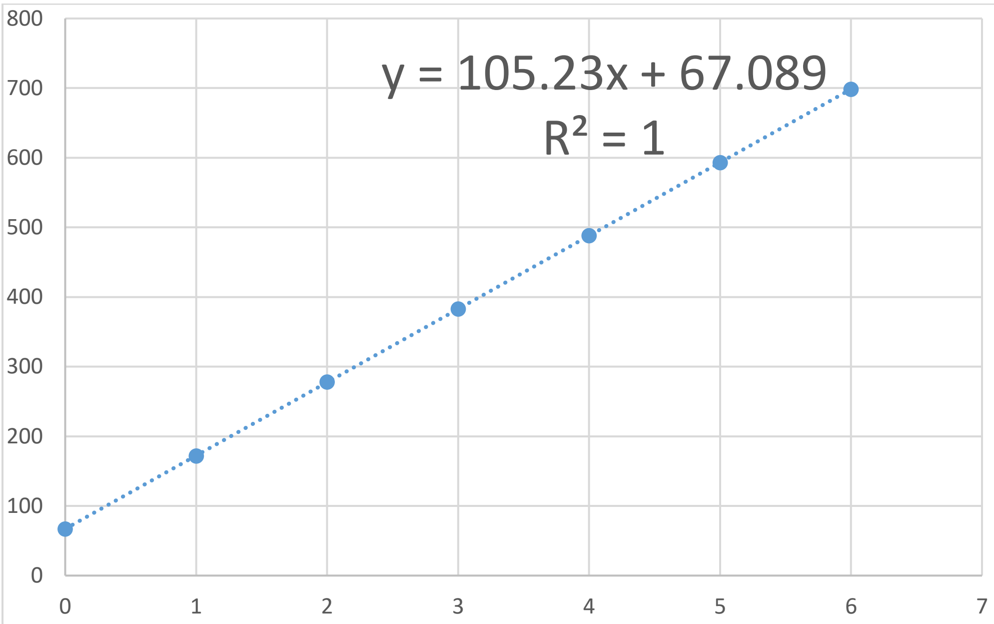


From $\gamma = 120°$ we get $\beta = 30°$, and using the equation we derived in A.4 we get:

$$n = \frac{1}{\sqrt{(\sin\beta)^2 + (\cos\beta)^2/(N-1)^2}} = 1.512$$

A.6: We will identify the beam exiting the disk after 4 refractions/reflections $(N = 4)$ and we will change the incident angle until we get δ_{min} for $N = 4$. We will measure $\alpha + k\gamma$ as a function of the number of times the beams hits the disk-air interface, k:

k	$\alpha + k\gamma$
0	67
1	172
2	278
3	383
4	488
5	593
6	698.5

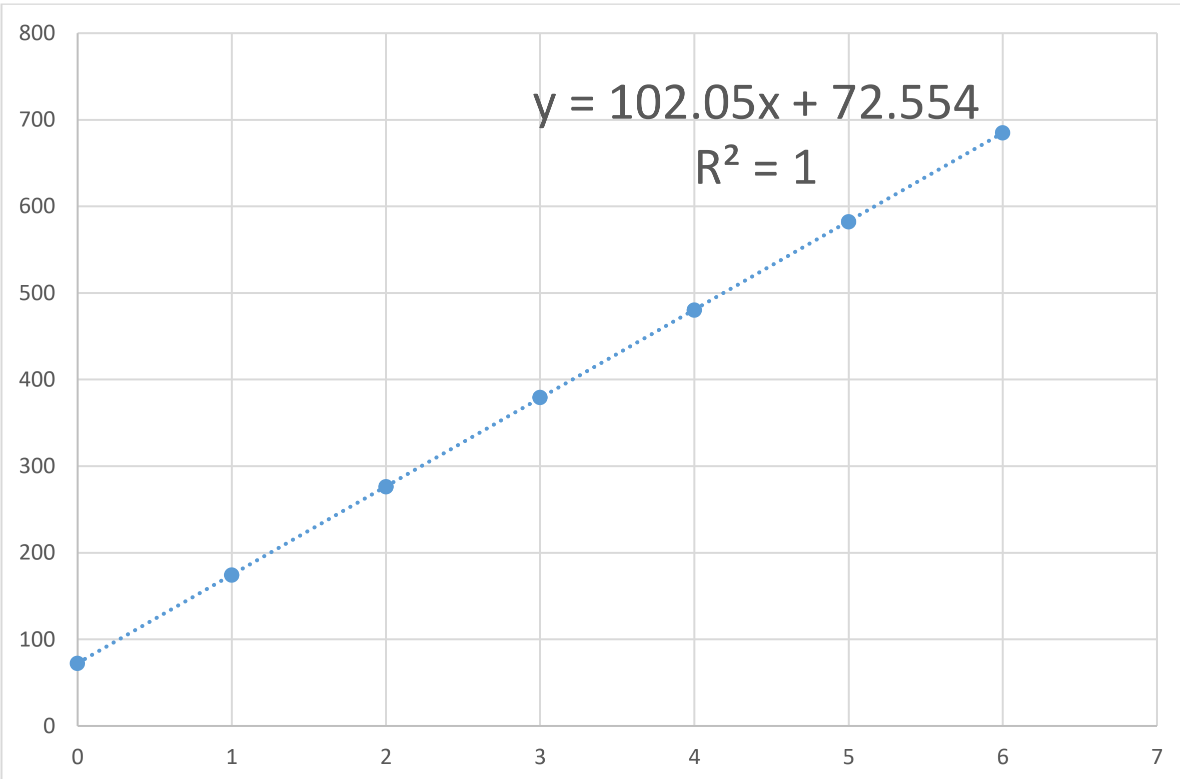


$$n = \frac{1}{\sqrt{(\sin\beta)^2 + (\cos\beta)^2/(N-1)^2}} = 1.511$$

We'll repeat this process for $N = 5$:

We will identify the beam exiting the disk after hitting the disk-air interface 5 times ($N = 5$) and measure $\alpha + k\gamma$ as a function of the number of hits, k:

k	$\alpha + k\gamma$
0	72.5
1	174.5
2	276.5
3	379.5
4	480.5
5	582.5
6	685

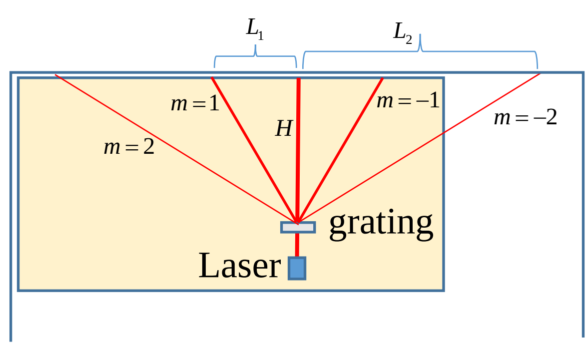


$$n = \frac{1}{\sqrt{(\sin\beta)^2 + (\cos\beta)^2/(N-1)^2}} = 1.519$$

Averaging the three results we get: $n = \dfrac{1.519+1.511+1.512}{3} = 1.514 \pm 0.004$

Section B – parameters of a diffraction grating

B.1: We will mark on the table a point Q, at a distance of about $H = 70cm$ from the screen – the wall of the experimental chamber - and at an equal distance from the chamber's side walls. Using the given measuring tape we will mark on the screen two points P_1 and P_2, at an equal distance of about 100cm from the left and form the right of the marked point Q. On the screen, we will mark a point P, placed in the middle of the interval P_1P_2. Then, we will aim a laser to go through the points QP. This beam will be perpendicular to the wall that will be used as a screen.



Standard method:
We will place the grating such that the beam passes through it. By gently rotating the grating we will make sure that diffraction ordered 1 and -1 as well as 2 and -2 will appear in symmetrically around the zero order point. Note that the position of the zero order on the screen does not depend on the angle α. In this situation it ok to assume that the incident angle of the beam on the grating is $\alpha = 0$.
As in the sketch, we will measure H, L_1 and L_2 and use the relation $d \sin\theta_m = m\lambda$.
The measured values are $2L_1 = 53.3\text{cm}$, $2L_2 = 163.5\text{cm}$ and $H = 60.8\text{cm}$.

For the first order we get $\frac{\lambda}{d} = 0.4015$. For the second order we get $\frac{\lambda}{d} = 0.4012$.

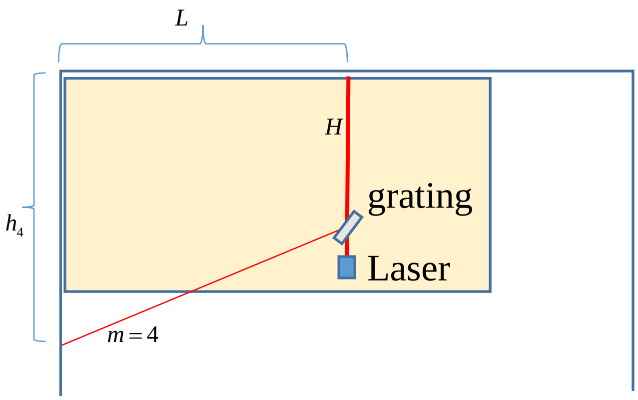
B.2: A second method
Getting higher orders is not possible at an incident angle of $\alpha = 0$. Thus we will change α and as a result the angle θ_m will change. There is an angle in which θ_m is minimal. By differentiating the relation $d\left(\sin\alpha + \sin(\theta_m - \alpha)\right) = m\lambda$ by α we get that at the minimum ($\frac{d\theta_m}{d\alpha} = 0$) one gets $\cos\alpha - \cos(\theta_m - \alpha) = 0 \Rightarrow \alpha = \frac{\theta_m}{2}$. From this we get $2d\sin(\frac{\theta_m}{2}) = m\lambda$.

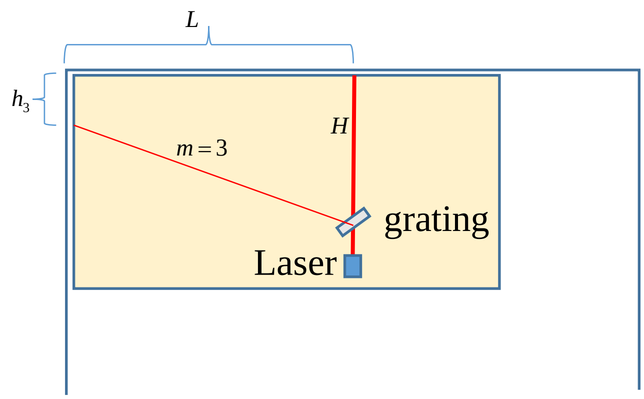
Note that there is no need to measure the angle α, but rather to identify, by changing α, the minimum of θ_m.

Using this method it is possible to measure also ordered $m = 1$ and $m = 2$. For $m = 2$ and $m = -2$ we can verify that the beam is perpendicular to the screen by making sure the distance of these two ordered from the zero order is identical.

For $m = 3$, we will change α to get θ_{3min} and measure the distances L and h_3.

The measured values, as shown in the sketch below, are $H = 67.0\,\text{cm}$, $L = 100.2\,\text{cm}$, $h_3 = 37.8\,\text{cm}$.





We get $\tan\theta_{3\min} = \frac{L}{H-h3} = \frac{100.2}{67.0-37.8} = 3.432$ and hence $\theta_{3\min} = 73.75°$

Therefore: $\frac{\lambda}{d} = \frac{2}{3}\sin\frac{\theta_{3\min}}{2} = \frac{2}{3}\sin\frac{73.75°}{2} = 0.400$

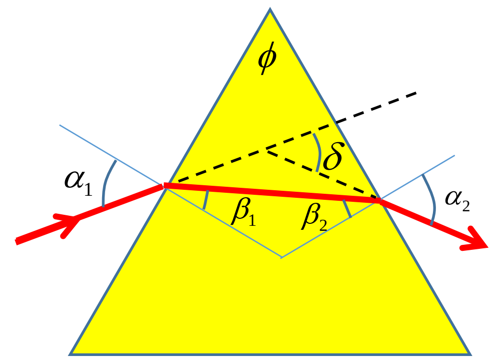
For $m = 4$ we will change α to get $\theta_{4\min}$ and measure the distance h_4.

The measured values are $H = 67.0\,\text{cm}$, $L = 100.2\,\text{cm}$, $h_4 = 96.3\,\text{cm}$.

From the sketch we get $\tan(\theta_{4\min} - 90°) = \frac{h_4 - H}{L} = \frac{96.3-67.0}{100.2} = 0.2924$

Hence $\theta_{4\min} = 106.3°$, therefore $\frac{\lambda}{d} = \frac{2}{4}\sin\frac{\theta_{4\min}}{2} = \frac{1}{2}\sin\frac{106.3°}{2} = 0.400$

Section C – the refraction index of a triangular prism



C.1: From the sketch showing the path of the laser beam and from the principle the beam path reversal we get that the deflection angle δ from the direction of in the incoming beam will not change if we switch the angles α_1 and α_2. Thus we get that δ achieves an extremum value (in fact, a minimal value) when the situation is perfectly symmetric, that is when $\alpha_1 = \alpha_2$. In this case,

$$\beta_1 = \beta_2 = \frac{\phi}{2}.$$

For the symmetric case, the incident angle α holds the relation $\alpha = \frac{\delta}{2} + \frac{\phi}{2}$ and from Snell's law we get $\sin\left(\frac{\delta}{2} + \frac{\phi}{2}\right) = n \sin\frac{\phi}{2}$.

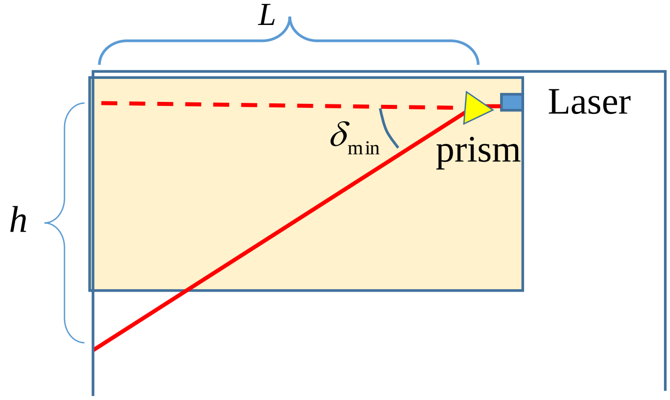
If the prism is not exactly equilateral, we will mark the angles of the prism by $\phi_i = 60^\circ + 2\varepsilon_i$. From the sum of angles in a triangle we get $\sum \varepsilon_i = 0$. Additionally $\beta_i = 30 + \varepsilon_i$. In this case $\delta_{\min} = \delta_0 + 2\Delta_i$ where δ_0 is the minimal δ when $\phi = 60^\circ$.

From Snell's law we get $\sin\left(\frac{\delta_0}{2} + 30^\circ + \Delta_i + \varepsilon_i\right) = n \sin(30^\circ + \varepsilon_i)$. Making the small angle approximation: $\sin\left(\frac{\delta_0}{2} + 30^\circ\right) + \cos\left(\frac{\delta_0}{2} + 30^\circ\right)(\Delta_i + \varepsilon_i) = n \sin 30^\circ + n \cos 30^\circ \cdot \varepsilon_i$

From the equation that holds for 60° prism we get $\cos\left(\frac{\delta_0}{2} + 30^\circ\right)(\Delta_i + \varepsilon_i) = n \cos 30^\circ \cdot \varepsilon_i$

Averaging for all three angles we get $\langle \Delta_i \rangle = 0$, and therefore $n = 2\sin\left(\frac{\langle \delta_{\min} \rangle}{2} + 30^\circ\right)$

C.2. We will use the full length of the table to magnify the distances as much as possible. We will build the setup, as described in the sketch, so that in the absence of the prism, the laser beam will hit the screen (the chamber's wall) perpendicularly. We will attach the prism holder base to the table using the adhesive tape. On it we will place the prism holder and the prism itself. We will rotate the prism to find the minimal deflection angle δ_{min}. We will then repeat the measurement of δ_{min} for each corner of the prism.



The measured values are given in the table:

Corner No.	L	h	$\delta_{\min}$
1	141.6 ± 0.2 cm	175.2 ± 0.3 cm	$51.05°\pm0.1°$
2	141.0 ± 0.2cm	167.1 ± 0.3cm	$49.84°\pm0.1°$
3	140.7 ± 0.2cm	171.4 ± 0.3cm	$50.62°\pm0.1°$

Calculation of the error in $\delta_{\min}$:

$$\tan\delta_{\min} = \frac{h}{L} \Rightarrow \frac{1}{\cos^2\delta_{\min}}\Delta\delta_{\min} = \sqrt{\left(\frac{\Delta h}{L}\right)^2 + \left(\frac{h\Delta L}{L^2}\right)^2}$$

Therefore, $\Delta\delta_{\min} = \cos^2\delta_{\min}\sqrt{\left(\frac{\Delta h}{L}\right)^2 + \left(\frac{h\Delta L}{L^2}\right)^2}$

Substituting the measured values we get

$$\Delta\delta_{\min} = \cos^2 51.05°\sqrt{\left(\frac{0.3}{141.6}\right)^2 + \left(\frac{175.2\cdot0.2}{141.6^2}\right)^2} = 0.0017\,rad = 0.1°$$

The error in the average value of the two angles is

$$\Delta\langle\delta_{\min}\rangle = \frac{0.1°}{\sqrt{3}} = 0.06° = 1\cdot10^{-3}\,rad$$

From the table we get that the average value of $\delta_{\min}$ is $\langle\delta_{\min}\rangle = 50.50°$

Therefore the refraction index of the prism is

$$n = 2\sin\left(\frac{\langle\delta_{\min}\rangle}{2} + 30°\right) = 2\sin\left(\frac{50.50°}{2} + 30°\right) = 2\sin 55.25° = 1.6433$$

And the error in n: $\Delta n = 2\cos 55.25° \cdot 0.5\Delta\langle\delta_{\min}\rangle = \cos 55.25° \cdot 1\cdot 10^{-3} = 6\cdot 10^{-4}$

Thus: $n = 1.6433 \pm 0.0006$

As the laser wavelength may vary between lasers up to a standard deviation of $\pm 10\,\text{nm}$, the value found in the literature is $n(\lambda \pm \Delta\lambda) = 1.6425 \pm 0.0007$.

Optical Measurements – Marking Scheme

Part A: The refractive index of a disk

A.1	drawing diagram:	
	The ruler, beam, disk, and α appear in the diagram	0.2 pts
	the incoming beam parallel to the diameter through $0°$	0.1 pts
	angle between the incoming beam and the ruler less than $25°$	0.1 pts
	at least 10 measurement points	0.3 pts
	full 15-75 degrees region	0.2 pts
	varying $\Delta\delta$ according to the spot size on the screen	0.1 pts
A.2	calculate β from δ and α for all rows in the table	0.1 pts
	calculate $\sin\alpha$ and $\sin\beta$ for all rows in the table	0.1 pts
	at least 8 measured points appear in the graph	0.1 pts
	the data covers at least 75% of each coordinate length	0.1 pts
	there are labels in each axis	0.1 pts
	plot regression line and calculate slope	0.1 pts
	value of n: if $1.50 \le n \le 1.53$	0.3 pts
	if $1.48 \le n < 1.50$ or $1.53 < n \le 1.55$	0.1 pts
	if $\Delta n \le 0.03$	0.1 pts
A.3	The graph includes a minimum angle of δ	0.1 pts
	labels in each axis and error bars of $\Delta\delta$ appear	0.1 pts
	value of $\delta_{\min}$: $336° \le \delta_{\min} \le 338°$	0.2 pts
	$335° \le \delta_{\min} < 336°$ or $338° < \delta_{\min} \le 339°$	0.1 pts
	value of $\alpha(\delta_{\min})$ $49° \le \alpha(\delta_{\min}) \le 51°$	0.1 pts
A.4	State that $\frac{d\delta}{d\alpha} = 0$	0.1 pts
	find that $\frac{d\beta}{d\alpha} = \frac{1}{N-1}$	0.1 pts

	conclude by Snell Law $\cos\alpha = \dfrac{n\cos\beta}{N-1}$	0.2 pts
	get $\dfrac{1}{n^2} = \sin^2\beta + \dfrac{\cos^2\beta}{(N-1)^2}$	0.3 pts
A.5	figure includes ray path and measured angles	0.1 pts
	measurement of $\phi_j = \alpha + j\gamma$ for $j = 0,1,2,3$	0.3 pts
	only for $j=0$ and $j=3$	0.2 pts
	only for $j=0$ and $j=2$	0.1 pts
	plot graph of ϕ_j vs. j	0.1 pts
	find value for β	0.1 pts
	value of n: $1.510 \le n \le 1.520$	0.2 pts
	$1.505 \le n < 1.510$ or $1.520 < n \le 1.525$	0.1 pts
A.6	For $N=4$:	
	measurement of $\phi_j = \alpha + j\gamma$ for $j = 0,1,2,...,6$ (7 values)	0.3 pts
	measurement of $\phi_j = \alpha + j\gamma$ for $j=0$ and $j=5$ or 6	0.2 pts
	measurement of $\phi_j = \alpha + j\gamma$ for $j=0$ and $j=3$	0.1 pts
	plot graph of ϕ_j vs. j	0.1 pts
	find value for β	0.1 pts
	value of n: $1.510 \le n \le 1.520$	0.2 pts
	$1.505 \le n < 1.510$ or $1.520 < n \le 1.525$	0.1 pts
	For $N=5$:	
	measurement of $\phi_j = \alpha + j\gamma$ for $j = 0,1,2,...,6$ (7 values)	0.3 pts
	measurement of $\phi_j = \alpha + j\gamma$ for $j=0$ and $j=5$ or 6	0.2 pts
	measurement of $\phi_j = \alpha + j\gamma$ for $j=0$ and $j = 4$	0.1 pts
	plot graph of ϕ_j vs. j	0.1 pts

	find value for β	0.1 pts
	value of n: $1.510 \le n \le 1.520$	0.2 pts
	$1.505 \le n < 1.510$ or $1.520 < n \le 1.525$	0.1 pts
	value of $\langle n \rangle$ and Δn: $1.512 \le \langle n \rangle \le 1.518$ and $\Delta n \le 0.01$	0.1 pts

Part B: The parameters of a diffraction grating

In part B, the final results of each student should be rescaled relative to the reference of $\lambda/d = 0.400$, according to the table supplied separately, using the ID of the grating recorded by the student in his/her answer sheet.

B.1	Plot the diagram with all requested items	0.1 pts
	Distance of the diffraction grating from the screen > 45 cm	0.1 pts
	for $m=1$ value of λ/d: $0.395 \le \lambda/d \le 0.405$	0.2 pts
	for $m=1$ value of λ/d: $0.39 \le \lambda/d < 0.395$ or $0.405 \le \lambda/d < 0.41$	0.1 pts
	for $m=2$ value of λ/d: $0.395 \le \lambda/d \le 0.405$	0.3 pts
	for $m=2$ value of λ/d: $0.39 \le \lambda/d < 0.395$ or $0.405 \le \lambda/d < 0.41$	0.1 pts
B.2	Plot the diagram with all requested items	0.1 pts
	the ray is definitely not perpendicular to the grating	0.1 pts
	The grating angle changes between $m=3$ and $m=4$	0.1 pts
	show minimum angle at $\alpha = \theta/2$	0.5 pts
	value of $\theta_{3\min}$: $73.0° \le \theta_{3\min} \le 74.5°$	0.3 pts
	for $m=3$ value of λ/d: $0.395 \le \lambda/d \le 0.405$	0.2 pts
	for $m=3$ value of λ/d: $0.39 \le \lambda/d < 0.395$ or $0.405 \le \lambda/d < 0.41$	0.1 pts
	value of $\theta_{4\min}$: $105.5° \le \theta_{4\min} \le 107.0°$	0.3 pts
	for $m=4$ value of λ/d: $0.395 \le \lambda/d \le 0.405$	0.2 pts

	for $m=4$ value of λ/d: $0.39 \le \lambda/d < 0.395$ or $0.405 \le \lambda/d < 0.41$	0.1 pts

Part C: The refractive index of a triangular prism

C.1	Understanding that $\delta_{\mathrm{min}} = \delta_{\mathrm{sym}}$	0.4 pts
C.2	Measuring $\delta_{\min}$ for at least one prism angle: $49.5° \le \delta_{\min} \le 51.5°$	0.3 pts
	Measuring $\delta_{\min}$ for two more prism angles: $49.5° \le \delta_{\min} \le 51.5°$	0.3 pts
	Distance between prism and screen larger than 120 cm	0.1 pts
	finding $\langle \delta_{\min} \rangle$: $50.30° \le \langle \delta_{\min} \rangle \le 50.70°$	0.3 pts
	making correct calculation of $\langle \Delta\delta_{\min} \rangle$, $\langle \Delta\delta_{\min} \rangle \le 0.1°$	0.1 pts
	finding n: $1.641 \le n \le 1.644$	0.4 pts
	$1.640 \le n < 1.641$ or $1.644 < n \le 1.645$	0.3 pts
	$1.639 \le n < 1.640$ or $1.645 < n \le 1.646$	0.2 pts
	$1.637 \le n < 1.639$ or $1.646 < n \le 1.648$	0.1 pts
	finding Δn using correct $\Delta\delta_{\mathrm{min}}$	0.1 pts

Wiedemann-Franz Law – Solution

Part A: Electrical conductivity of metals (1.5 points)

A.1 (1.0 points)

Magnet descend time:

Number	Copper [s]	Aluminum[s]	Brass [s]
1	17.77	9.23	6.1
2	17.96	9.39	5.83
3	18.16	9.22	6.04
4	18.15	9.37	5.86
5	17.76	9.36	6.16
6	18.2	9.44	5.92
7	17.67	9.65	5.9
8	17.9	9.18	6.08
9	17.67	9.41	5.86
10	18.36	8.96	5.99
Average	17.96	9.32	5.97

A.2 (0.5 points)

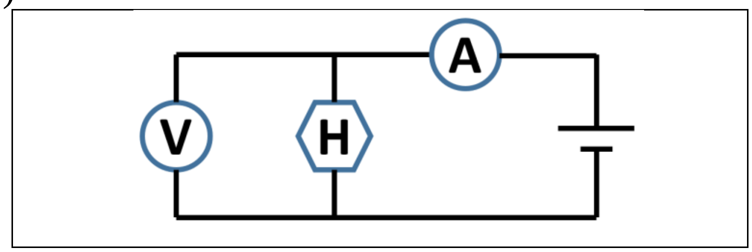
	Copper	Aluminum	Brass
Electrical conductivity $\left[\frac{1}{\Omega m}\right]$	5.97×10^7	2.98×10^7	1.60×10^7

Part B: Thermal conductivity of copper (3.0 points)

B.1 (0.1 points)

Rod 1 temperature : **22.76 [C]**

B.2 (0.5 points)



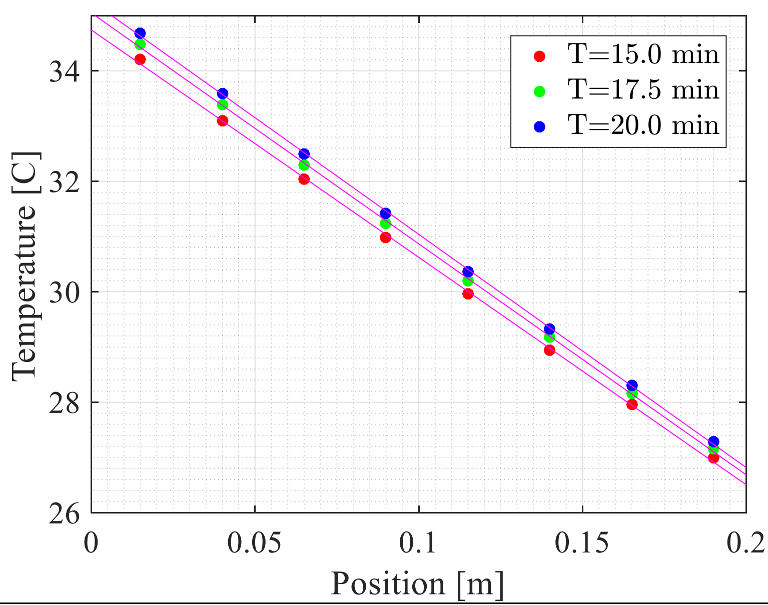
B.3 (0.1 points)

$P = I \cdot V = 5.51[\mathrm{W}]$

B.4 (0.5 points)

Time [S]	T1 [C]	T2 [C]	T3 [C]	T4[C]	T5 [C]	T6 [C]	T7 [C]	T8 [C]
900	26.98	27.96	28.95	29.96	30.98	32.03	33.10	34.20
1050	27.16	28.16	29.17	30.20	31.240	32.30	33.38	34.48
1200	27.29	28.30	29.33	30.37	31.42	32.49	33.58	34.68

B.5 (1.0 points)



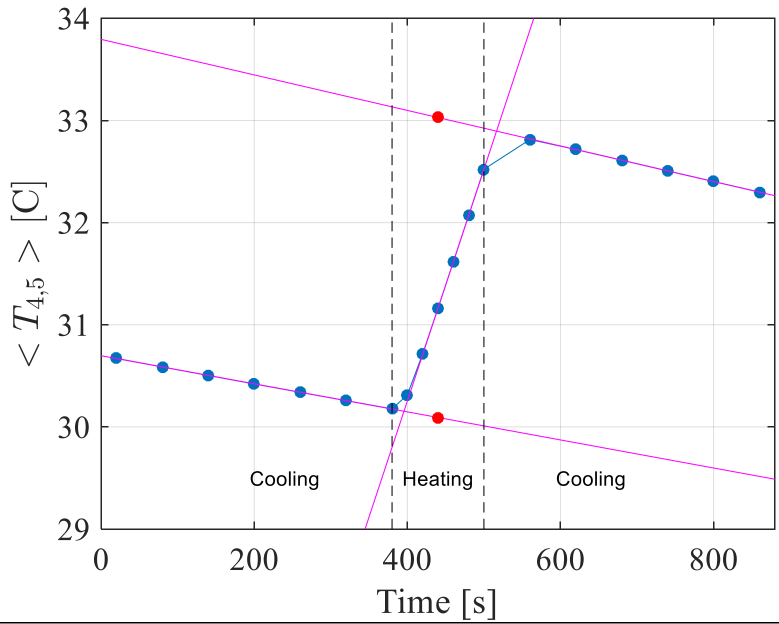
B.6 (0.5 points)
$$\kappa_0 = -\frac{P}{A\frac{\Delta T}{\Delta x}} = -\frac{5.51[W]}{\pi\cdot\left(10^{-2}[m]\right)^2\cdot\left(-41.8\left[\frac{K}{m}\right]\right)} = 420\left[\frac{W}{mK}\right]$$ $$\frac{\Delta T}{\Delta t} = \frac{31.04[C]-30.62[C]}{5\cdot 60[s]} = 1.4\cdot 10^{-3}\left[\frac{K}{s}\right]$$
B.7 (0.3 points) **<u>higher value</u>** We expect a **<u>higher value</u>** of κ_0 compared with the real κ_{cu} because of 2 reasons: 1. A part of the supplied heat power is lost through the side walls. Therefore, the heat transfer through the cross-section of the rod is smaller. 2. Since the system is not in a steady state ($\frac{\Delta T}{\Delta t} \neq 0$), the corresponding power involved should be subtracted from the power supplied by the heater.

Part C: Heat loss and heat capacity of copper (4.0 points)

C.1 (1.0 points)

$Time[s]$	$T_1[C]$	$T_2[C]$	$T_3[C]$	$T_4[C]$	$T_5[C]$	$T_6[C]$	$T_7[C]$	$T_8[C]$	$T_{av}[C]$
20				30.67	30.67				30.67
80				30.59	30.59				30.59
140				30.50	30.50				30.50
200				30.42	30.42				30.42
260				30.34	30.34				30.34
320				30.26	30.26				30.26
380				30.18	30.18				30.18
400				30.38	30.25				30.31
420				30.87	30.56				30.72
440				31.37	30.96				31.16
460				31.85	31.38				31.61
480				32.32	31.82				32.07
500				32.78	32.26				32.52
560				32.88	32.75				32.81
620				32.73	32.70				32.72
680				32.61	32.61				32.61
740				32.51	32.51				32.51
800				32.40	32.40				32.40
860				32.30	32.30				32.30

C.2 (1.0 points)



C.3 (1.0 points)

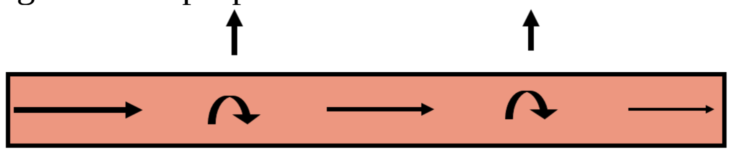
The purpose of this part is to correct to first order the result in part B. Hence, every solution within 10% accuracy is accepted (see marking scheme).

Solution 1 (using slopes):	Solution 2 (using jump):
$P_{loss} = c_p \cdot m \cdot \left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling}$	$P_{loss} = c_p \cdot m \cdot \left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling}$
$P_{in} = c_p \cdot m \cdot \left(\left.\frac{\partial T_{av}}{\partial t}\right\|_{Heating} - \left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling} \right)$	$P_{in} \cdot \Delta t = c_p \cdot m \cdot \Delta T$
Where $\left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling}$ is the average of both cooling slopes.	Where $\left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling}$ is the average of the two cooling slopes, and ΔT is the extrapolated jump in temperature half way though the heating time interval.
$c_p \cdot m = \frac{5.5[W]}{\left(2.27 \cdot 10^{-2}\left[\frac{K}{s}\right] + 1.6 \cdot 10^{-3}\left[\frac{K}{s}\right]\right)}$	$c_p \cdot m = \frac{P_{in} \cdot \Delta t}{\Delta T} = \frac{5.5[W] \cdot 120[s]}{2.94[K]} = 224\left[\frac{J}{K}\right]$

$c_p \cdot m = 226\left[\frac{J}{K}\right] \Rightarrow c_p = 390\left[\frac{J}{kg \cdot K}\right]$ Which is 1% off the correct value. $P_{loss} = 226\left[\frac{J}{K}\right] \cdot 1.4 \cdot 10^{-3}\left[\frac{K}{s}\right] = 0.32[W]$	$c_p = 386\left[\frac{J}{kg \cdot K}\right]$ which is the correct value. $P_{loss} = 224\left[\frac{J}{K}\right] \cdot 1.4 \cdot 10^{-3}\left[\frac{K}{s}\right] = 0.31[W]$

C.4 (1.0 points)

The temperature gradient is proportional to the local heat flow.



To first order, the average temperature gradient will be proportional to the average heat flow. Therefore, the temperature gradient will be proportional to $P_{in} - \frac{1}{2}P_{losses}$:

$$\kappa = \frac{P_{in} - \frac{1}{2}P_{absorb} - \frac{1}{2}P_{loss}}{A \cdot (\Delta T / \Delta x)} = \frac{P_{in} - \frac{1}{2}c_p \cdot m \cdot \frac{\Delta T}{\Delta t} - \frac{1}{2}\dot{Q}_{loss}}{A \cdot \Delta T / \Delta x} = \kappa_0 \cdot \frac{P_{in} - \frac{1}{2}c_p \cdot m \cdot \frac{\Delta T}{\Delta t} - \frac{1}{2}\dot{Q}_{loss}}{P}$$

$$\kappa = 420\left[\frac{W}{mK}\right] \cdot \frac{5.51[W] - \frac{1}{2} \cdot 226\left[\frac{J}{K}\right] \cdot 1.4 \cdot 10^{-3}\left[\frac{K}{s}\right] - \frac{1}{2} \cdot 0.32[W]}{5.51[W]} = 396\left[\frac{W}{mK}\right]$$

Which gives an error of 2.5% error compared to expected $385\left[\frac{W}{mK}\right]$. We expect a 1% systematic error (see appendix).

Part D: Thermal conductivity of multiple metals (1.0 points)

D.1 (0.1 points)

$T = 22.65[C]$

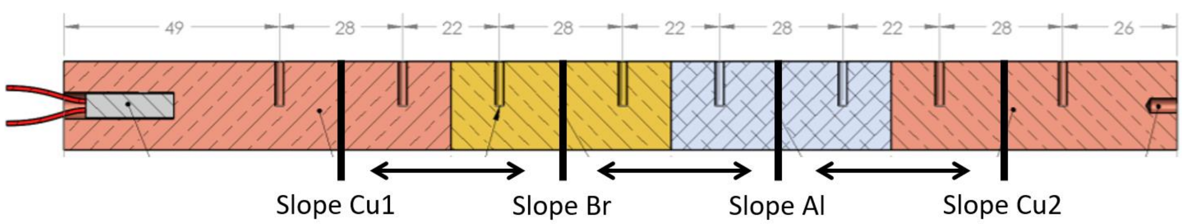
D.2 (0.2 points)

Time of measurement: $1041[s]$

$T_1[C]$	$T_2[C]$	$T_3[C]$	$T_4[C]$	$T_5[C]$	$T_6[C]$	$T_7[C]$	$T_8[C]$
41.68	40.51	38.51	34.65	32.47	30.71	29.63	28.62

$\Delta T_{cu1}/\Delta x$	$\Delta T_{Br}/\Delta x$	$\Delta T_{Al}/\Delta x$	$\Delta T_{cu2}/\Delta x$
$41.79\left[\frac{K}{m}\right]$	$137.86\left[\frac{K}{m}\right]$	$62.86\left[\frac{K}{m}\right]$	$36.07\left[\frac{K}{m}\right]$

D.3 (0.7 points)



$$\kappa_{Brass} = \kappa_{Copper} \cdot \frac{\frac{2}{3}(\Delta T_{cu1}/\Delta x)+\frac{1}{3}(\Delta T_{cu2}/\Delta x)}{\Delta T_{Br}/\Delta x} = 115\left[\frac{W}{mK}\right]$$

$$\kappa_{Aluminum} = \kappa_{Copper} \cdot \frac{\frac{1}{3}(\Delta T_{cu1}/\Delta x)+\frac{2}{3}(\Delta T_{cu2}/\Delta x)}{\Delta T_{Al}/\Delta x} = 239\left[\frac{W}{m \cdot K}\right]$$

Part E: The Wiedemann-Franz law (0.5 points)

E.1 (0.5 points)

	Copper	Aluminum	Brass
$\sigma\ [\Omega^{-1}m^{-1}]$ Electric conductivity	5.97×10^7	2.98×10^7	1.60×10^7
$\kappa\ \left[\frac{W}{Km}\right]$ Heat conductivity	396	239	115
$L\ \left[\frac{W\Omega}{K^2}\right]$ Lorenz coefficient	2.21×10^{-8}	2.67×10^{-8}	2.40×10^{-8}

Wiedemann-Franz Law – Marking Scheme

Part A: Electric conductivity of metals (1.5 points)

A.1	Measuring magnet fall (1.0 pts)	
	The number of total measurements : if $N \leq 15$	0.2 pts
	if $15 < N \leq 21$	0.5 pts
	if $N > 21$	0.7 pts
	Average travel time within 10% of solution for 2 out of 3 rods	0.3 pts
A.2	Calculation of conductivity (0.5 pts)	
	Correct calculation of conductivity from A1	0.1 pts
	Final result for 2 out of 3 values: Within 10% of correct value	0.4 pts
	Within 20% of correct value	0.2 pts

Part B: Thermal conductivity of copper (3.0 points)

B.1	Writing room temperature with units	0.1 pts
B.2	Design a 4-probe circuit (0.5 pts)	
	Drawing ammeter in series with source and heater	0.2 pts
	Measuring voltage on heater and not power source	0.3 pts
B.3	Writing the equation for power and proper calculation	0.1 pts
B.4	Writing thermometers readings (0.5 pts)	
	Complete set (24 temperatures in table)	0.2 pts
	Units	0.1 pts
	2 digits after decimal point	0.1 pts
	Times within 1 minute of requirement (15,17.5,20 minutes)	0.1 pts
B.5	Thermal equilibrium graph (1.0 pts)	
	All 24 points are plotted	0.4 pts
	Correct axes, with units	0.2 pts

	Points span on 1/2 the area of graph paper	0.2 pts
	Slope is sketched for 17.5 min	0.2 pts
B.6	Obtaining κ_0 (0.5 points)	
	Correct expression for κ_0	0.1 pts
Op.1	Range of κ_0 $\left[W/\left(mK\right)\right]$: $404 \le \kappa_0 \le 446$	0.2 pts
	$382 \le \kappa_0 \le 468$	0.1 pts
	Range of $\Delta T/\Delta t\left[K/s\right]$: $1.25 \cdot 10^{-3} \le \Delta T/\Delta t \le 1.55 \cdot 10^{-3}$	0.2 pts
	$1.1 \cdot 10^{-3} \le \Delta T/\Delta t \le 1.7 \cdot 10^{-3}$	0.1 pts
Op.2	The value of the corrected κ (using the method in the solution) with κ_0 ,$\Delta T/\Delta t$ and c_p, P_{loss} from the official solution is in range:	
	$376 \le \kappa \le 416$	0.4
	$356 < \kappa < 376$ or $416 < \kappa < 436$	0.2
B.7	Correct answer - **Higher value**	0.3 pts

Part C: Heat loss and heat capacity of copper (4.0 points)

C.1	Cooling-Heating-Cooling cycle (1.0 pts)	
	Number of measurement points for each step: if $3 \le N < 5$	0.1 pts
	if $N \ge 5$	0.2 pts
	Heating step time in range $1\left[\min\right] \le t \le 3\left[\min\right]$	0.2 pts
	Cooling steps time $t > 200\left[s\right]$	0.2 pts
	If average between T4,T5 or average over all thermometers	0.2 pts
	Used only T4 or only T5	0.1 pts
	The reported temperature mid-heating is:	
	Less than 2.5 [C] away from average temperature in B.4	0.2 pts
	Between 2.5[C] and 4.0[C] from average temperature in B.4	0.1 pts

C.2	Cooling – Heating – Cooling graph (1.0 pts)	
	Correct axes, units on axes	0.2 pts
	Number of points on graph: $N \geq 15$	0.4 pts
	$12 \leq N < 15$	0.2 pts
	Points span on 1/2 the area of graph paper	0.2 pts
	Slope lines are plotted for cooling steps	0.2 pts
C.3	Obtaining c_p and P_{loss} (1.0 pts)	
	$P_{loss} = c_p \cdot m \cdot \left.\frac{\partial T_{av}}{\partial t}\right\vert_{Cooling}$	0.2 pts
	$P_{in} = c_p \cdot m \cdot \left(\left.\frac{\partial T_{av}}{\partial t}\right\vert_{Heating} - \left.\frac{\partial T_{av}}{\partial t}\right\vert_{Cooling} \right)$ or $P_{in} \cdot \Delta t = c_p \cdot m \cdot \Delta T$	0.4 pts
	Range of c_p in $\left[J/(kg \cdot K)\right]$: $425 \leq c_p \leq 350$	0.2 pts
	$465 \leq c_p \leq 310$	0.1 pts
	Range of P_{loss} in $[W]$: $0.25 \leq P_{loss} \leq 0.38$	0.2 pts
	$0.19 \leq P_{loss} \leq 0.44$	0.1 pts
C.4	Correct κ (1.0 pts)	
	$c_p \cdot m \cdot \frac{\Delta T}{\Delta t}$	0.1 pts
	$c_p \cdot m \cdot \frac{\Delta T}{\Delta t}$ and P_{loss} are treated the same way	0.1 pts
	Form of equation $\kappa = \frac{\kappa_0}{P}\left(P - \alpha \cdot \left(c_p \cdot m \cdot \frac{\Delta T}{\Delta t} + P_{loss}\right)\right)$	0.2 pts
	Writing that $\alpha = 0.5$	0.3 pts
	κ range in $\left[W/(mK)\right]$: $376 \leq \kappa \leq 416$	0.3 pts
	$356 < \kappa < 376$ or $416 < \kappa < 436$	0.2 pts

Part D: Thermal conductivity of multiple metals (1.0 points)

D.1	Writing temperature with units	0.1 pts
D.2	Temperature measurements (0.2 pts)	
	Measurement time is greater than 15 minutes	0.1 pts
	Correct calculation of $\Delta T / \Delta x$ using $28mm$ spacing	0.1 pts
D.3	Calculation of κ for other metals (0.7 pts)	
	general form of $\kappa_\alpha = \kappa_{copper} \cdot \frac{Slope}{(\Delta T / \Delta x)_\alpha}$	0.1 pts
	Weighted average: 1:2 and 2:1 average between coppers (correct direction, see solution)	0.4 pts
	Weighted average but wrong weights	0.2 pts
	Slope from closest copper or simple average	0.1 pts
	$103[W/(mK)] \le \kappa_{brass} \le 126[W/(mK)]$	0.1 pts
	$215[W/(mK)] \le \kappa_{Aluminum} \le 263[W/(mK)]$	0.1 pts

Part E: The Wiedemann-Franz law (0.5 points)

E.1	Wiedemann-Franz law table (0.5 pts)	
	Calculation of Lorenz number, using absolute temperature	0.1 pts
	$2.12[W\Omega / K^2] \le L_{copper} \le 2.39[W\Omega / K^2]$	0.2 pts
	$2.13[W\Omega / K^2] \le L_{Brass} \le 2.71[W\Omega / K^2]$	0.1 pts
	$2.00[W\Omega / K^2] \le L_{Aluminum} \le 2.54[W\Omega / K^2]$	0.1 pts

Please note that this marking scheme might change, particularly the ranges.