1 Twee prisma's (4pt)

Als het geheel begint te schuiven moet het totale moment nul blijven in het systeem van de tafel. Dus horizontaal geldt: $m \cdot v_{k,x} = 4m \cdot v_g$, dus $v_{k,x} = 4v_g$, Dus $v_{\text{rel,x}} = 5v_g$.

dan kun je schrijven: $v_{rel} = v_{rel,x}/\cos 30 = 5v_g/\left(\frac{1}{2}\sqrt{3}\right)$ en dus geldt: $v_g = \frac{\sqrt{3}}{10} v_{rel}$.

2 Vallende draadlus. (Giancoli 29.79) (5pt)

(a) As the loop falls out of the magnetic field, the flux through the loop decreases with time creating an induced emf in the loop. The current in the loop is equal to the emf divided by the resistance, which can be written in terms of the resistivity:

$$I = \frac{\varepsilon}{R} = \left(\frac{\pi d^2/4}{\rho 4l}\right)\frac{d\Phi_B}{dt} = \left(\frac{\pi d^2}{16\rho l}\right)B\frac{dA}{dt} = \frac{\pi d^2}{16\rho l}Blv = \frac{\pi d^2}{16\rho}Bv$$

This current induces a force on the three sides of the loop in the magnetic field. The forces on the two vertical sides are equal and opposite and therefore cancel.

$$F = IlB = \frac{\pi d^2}{16\rho}BvlB = \frac{\pi d^2 B^2 lv}{16\rho}$$

By Lenz's law this force is upward to slow the decrease in flux.

(b) Terminal speed will occur when the gravitational force is equal to the magnetic force.

$$F_g = \rho_m\left(4\pi l\frac{d^2}{4}\right)g = \frac{\pi d^2 B^2 l v_T}{16\rho} \rightarrow v_T = \frac{16\rho\rho_m g}{B^2}$$

3 Duikplank. (Giancoli 14.80) (3pt)

For the pebble to lose contact with the board means that there is no normal force of the board on the pebble. If there is no normal force on the pebble, then the only force on the pebble is the force of gravity, and the acceleration of the pebble will be *g* downward, the acceleration due to gravity. This is the maximum downward acceleration that the pebble can have. Thus if the board's downward acceleration exceeds *g*, then the pebble will lose contact. The maximum acceleration and the amplitude are related by $a_{\max} = 4\pi^2 f^2 A$.

$$a_{\max} = 4\pi^2 f^2 A \le g \quad \rightarrow \quad A \le \frac{g}{4\pi^2 f^2} \le \frac{9.80\,\text{m/s}^2}{4\pi^2(2.5\text{ Hz})^2} \le \boxed{4.0\times10^{-2}\,\text{m}}$$

4 Echo. (SNON 1995) (4pt)

Het verschijnsel berust op interferentie. De muur gedraagt zich als een reflectie-tralie. De klap is een korte puls waarin een heel spectrum van frequenties voorkomt. Het deel van de muur recht tegenover de toehoorder weerkaatst alle golven. De hoek waaronder de maximale intensiteit van het geluid bij een bepaalde frequentie wordt waargenomen is golflengte afhankelijk. De golven zijn, afkomstig van de verschillende ribbels, in fase als:

$$\frac{\lambda/2}{d} = \sin\alpha$$

De tijd dat een golf die in het centrum weerkaatst wordt, weer bij de waarnemer aankomt, is:

$$t = \frac{2L}{c}$$

Voor een golf die onder een hoek wordt waargenomen geldt:

$$t = \frac{2L}{c}\frac{1}{\cos\alpha}$$

Met $\lambda \cdot f = c$ volgt nu:

$$t = \frac{2L}{c}\frac{2df}{\sqrt{4d^2 f^2 - c^2}}$$

De hoogste frequenties komen dus het eerst aan; er is overigens een ondergrens aan de frequenties:

$$f_0 = \frac{c}{2d}$$

5 Kringproces. (Giancoli 20.69) (6pt)

All of the processes are either constant pressure or constant volume, and so the heat input and output can be calculated with specific heats at constant pressure or constant volume. This tells us that heat is input when the temperature increases, and heat is exhausted when the temperature decreases. The lowest temperature will be the temperature at point b. We use the ideal gas law to find the temperatures.

$$PV = nRT \quad\rightarrow\quad T = \frac{PV}{nR} \quad\rightarrow$$

$$T_b = \frac{P_0V_0}{nR},\ T_a = \frac{P_0(2V_0)}{nR} = 2T_b,\ T_c = \frac{(3P_0)V_0}{nR} = 3T_b,\ T_d = \frac{(3P_0)(2V_0)}{nR} = 6T_b$$

Process ab: $W_{ab} = P\Delta V = P_0(-V_0) = -P_0V_0\ ;\ Q_{ab} < 0$

Process bc: $W_{bc} = P\Delta V = 0\ ;\ Q_{bc} = nC_V\Delta T = \tfrac{3}{2}nR(T_c - T_b) = \tfrac{3}{2}nR(2T_b) = \tfrac{3}{2}nR\left(2\frac{P_0V_0}{nR}\right) = 3P_0V_0$

Process cd: $W_{bc} = P\Delta V = 3P_0V_0\ ;$

$Q_{cd} = nC_P\Delta T = \tfrac{5}{2}nR(T_d - T_c) = \tfrac{5}{2}nR(3T_b) = \tfrac{5}{2}nR\left(3\frac{P_0V_0}{nR}\right) = \tfrac{15}{2}P_0V_0$

Process da: $W_{da} = P\Delta V = 0\ ;\ Q_{da} < 0$

$$e_{\text{rectangle}} = \frac{W}{Q_H} = \frac{3P_0V_0 - P_0V_0}{3P_0V_0 + \tfrac{15}{2}P_0V_0} = \frac{2}{\tfrac{21}{2}} = 0.1905 \approx \boxed{0.19}$$

6 Vliegtuig vleugel (IPhO 2012, Estland) (3pt)

In the plane's reference frame, along the channel between two streamlines the volume flux of air (volume flow rate) is constant due to continuity. The volume flux is the product of speed and channel's cross-section area, which, due to the two-dimensional geometry, is proportional to the channel width and can be measured from the figure. Due to the absence of wind, the unperturbed air's speed in the plane's frame is just v_0.

Measuring the dimensions in front of the wing (a = 5,5 mm) and at point P (b = 7,0 mm) we can write:

$$v_0 a = ub$$

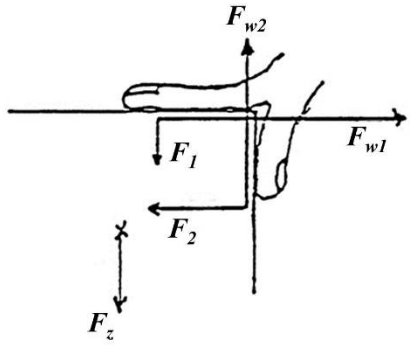
And hence

$$u = v_0\frac{a}{b}$$

Since at point P, the streamlines are horizontal where all the velocities are parallel, the vector addition is reduced to the scalar addition: the air's ground speed

$$v_P = v_0 - u = v_0\left(1 - \frac{a}{b}\right) = 100\left(1 - \frac{5,5}{7,0}\right) = 21\,\text{m/s}$$

7 Fles op de kop. (Oude 1e ronde opgave) (4pt)



Voor de wrijvingscoëfficiënt geldt (uit de tekst):

$$F_W = f \cdot F$$

In de verticale richting geldt:

$$F_z + F_1 = F_{W2} = f \cdot F_2$$

In de horizontale richting geldt:

$$F_2 = F_{W1} = f \cdot F_1$$

Invullen van de verticale vergelijking in die van de horizontale

$$F_z + F_1 = F_{W2} = f \cdot F_2 = f\left(f \cdot F_1\right) = f^2 \cdot F_1$$

Dit levert:

$$F_z = \left(f^2 - 1\right) \cdot F_1$$

Dit kan alleen als geldt: $f^2 > 1 \rightarrow f > 1$

8 Rollend balletje. (Singapore 2 november 1991) (4pt)

Op het balletje werken drie krachten: de zwaartekracht (*W*), de normaalkracht (*N*) en de wrijvingskracht (*f*).

In de richting van de beweging geldt m.b.v. de 2^e wet van Newton:

$$Mg \sin 30° - f = Ma \qquad [1]$$

Voor de rotatie van de bal (t.o.v. het middelpunt) geldt $\tau = fR = I\alpha$. Hierin is α de hoekversnelling. Deze is gekoppeld aan de versnelling a middels $a = R\alpha$. Omdat de bal zonder slippen rolt, moet wel gelden:

$$f = \frac{Ia}{R^2} \qquad [2]$$

We kunnen de wrijvingskracht f elimineren uit [1] en [2]:

$$\tfrac{1}{2} Mg = \left(M + \frac{I}{R^2}\right) a \qquad [3]$$

Het traagheidsmoment *I* kan geschreven worden als $I = kMR^2$ waarin $0 < k < 1$. (Voor een uniforme homogene bal geldt bijvoorbeeld $k = \frac{2}{3}$, voor een homogene schil $k = 1$.)

Formule [3] is nu te herschrijven als:

$$\tfrac{1}{2} Mg = \left(1 + k\right) Ma \qquad [4]$$

Oftewel:

$$a = \frac{g}{2\left(1+k\right)}$$

Omdat geldt dat $k < 1$ moet wel gelden dat:

$$a > \frac{g}{4} \approx 2,4 \text{ m/s}^2$$

Een gemeten waarde van 2,0 m/s² is dus niet mogelijk.

Invullen van waarden in [4]:

$$k = \frac{g}{2a} - 1 = 0,53$$

$$I = 0,53 \times 0,40 \times \left(0,05\right)^2 = 5,3 \cdot 10^{-4} \text{kgm}^2$$

9 Opladen. (200 Puzzling Physics Problems, P155) (5pt)

(a) De condensator heeft na het opladen een opgeslagen energie:

$$E = \tfrac{1}{2}C\left(2\varepsilon\right)^2 = 2C\varepsilon^2$$

Voor de weerstand R geldt voor het gehele oplaadproces:

$$E = \int Pdt = \int U \cdot I = \int I^2 R dt = R\int_0^\infty I_0^2 e^{-2t/RC} dt$$

Maar hierin wordt I_0 gegeven door $2\varepsilon/R$ zodat volgt voor de energie:

$$E = R\int_0^\infty I_0^2 e^{-2t/RC} dt = R\frac{4\varepsilon^2}{R^2}\left(-\frac{RC}{2}e^{-2t/RC}\Big|_0^\infty\right) = R\frac{4\varepsilon^2}{R^2}\frac{RC}{2} = 2\varepsilon^2 C$$

Daarmee is aangetoond dat de opgeslagen energie in de condensator inderdaad even groot is als de gedissipeerde energie in de weerstand R.

(b) Stel dat er in de eerste situatie, om de condensator op te laden tot een spanning 2ε, een lading $2Q$ gestroomd heeft. De batterijen hebben dan in totaal geleverd:

$$E = 2\varepsilon \cdot 2Q = 2\varepsilon \cdot 2C\varepsilon = 4C\varepsilon^2$$

Er is in hierboven aangetoond dat de helft hiervan gedissipeerde energie in de weerstand R is. Om de condensator met 1 batterij op te laden tot spanning ε heeft er een lading Q gestroomd. Voor deze batterij geldt dus voor de geleverde energie:

$$E = \varepsilon \cdot Q = \varepsilon \cdot C\varepsilon = C\varepsilon^2$$

Vervolgens laden we condensator verder op. Er stroomt nu weer een lading Q maar nu met een spanningsverschil 2ε. De geleverde energie wordt dan:

$$E = 2\varepsilon \cdot Q = 2\varepsilon \cdot C\varepsilon = 2C\varepsilon^2$$

In totaal wordt dat dus voor het gehele oplaadproces:

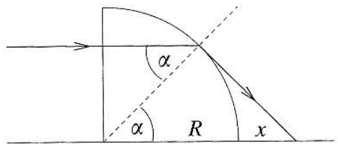
$$E = C\varepsilon^2 + 2C\varepsilon^2 = 3C\varepsilon^2$$

De condensator heeft een opgeslagen energie van $2C\varepsilon^2$ dus moet er wel in de weerstand R een energie gedissipeerd zijn van $E = 4C\varepsilon^2 + 3C\varepsilon^2 = C\varepsilon^2$ Dat is minder dan in het geval van het direct opladen met de twee batterijen in serie.

(Dit probleem kan veralgemeniseerd worden, zie Problem155 uit 200 Puzzling Physics Problems)

10 Cilinder prisma. (200 Puzzling Physics Problems, P55) (4pt)

Consider the light beam as consisting of parallel light rays. They cross the vertical plane face of the quarter-cylinder without changing their direction, and strike the curved surface of the cylinder at various angles of incidence. The normals at the points of incidence of the rays are radii of the cylinder. The higher the position of a light ray entering the quarter-cylinder, the larger is its angle of incidence at the cylinder's curved surface. The angle of incidence for the ray shown in the figure is the critical angle for total internal reflection. Therefore only light rays closer to the table than this one can leave the quarter-cylinder (refracted to different extents). The limiting case is determined using the figure:



$\sin\alpha_h = \frac{1}{n} = \frac{2}{3}$ and $\frac{R}{R+x} = \cos\alpha_h$

which yield x = 1,71 cm. This is the closest to the quarter-cylinder that light can reach the table.

As the angle of incidence of light rays close to the table top is smaller, they are deviated less from their original direction by refraction, and therefore might reach the surface of the table further away. One is inclined to think that, in principle, the light patch could reach to any distance along the table, since the direction of a light ray travelling adjacent to the surface of the table is not altered. This, however, is false; the path of each light ray can be parameterised (e.g. as a function of the angle of incidence), and it can then be shown that each ray does not get very far up the table.

Instead of through tedious calculation, the furthest point of the light patch can be found by means of a simple 'trick'. Consider the part of the quarter cylinder close to the table as a plano-convex lens. The cylinder material before the lens behaves like a plano-parallel plate and can be ignored. The focal length of the plano-convex lens can be calculated using the thin lens formula:

$$\frac{1}{f} = \frac{n-1}{R}$$

This yields f = 10 cm, and this is the distance from the quarter-cylinder of the furthest point of the light patch.

11 Lijnenspectrum. (200 Puzzling Physics Problems, P127) (3pt)

$$n\lambda = d\sin\alpha = \frac{10^{-3}\,\text{m}}{300}\sin(24{,}46°) = 1380\ \text{nm}$$

The only possible values for n and λ to put the red and blue/violet light into the appropriate parts of the spectrum are n = 2, λ = 690 nm and n = 3, λ = 460 nm.

In all physically possible cases

$$n\lambda \le d\sin 90° = 3333\ \text{nm}$$

and the only other pair of integers which are in the ratio *3m* : *2m,* with *m* less than (3333/1380) = 2.4, is 6 and 4. Thus there is only one more angle at which a two-component line will be observed; i.e. at

$$\sin^{-1}\frac{(6\times 460)}{3333} = 55{,}9°$$

12 Holte. (Oude som uit materiaal Hans Jordens) (5pt)

Let the compressed air push an imaginary piston along a tube. The total energy stored in the cavity will be given up to the piston when the inside pressure p equals the outside pressure p_0 at which time the piston is $x = x_{\max}$.

If the piston area is A then the force on the piston is $(p - p_0)A$. We thus get:

$$W_{stored} = \int_0^{x_{\max}} (p - p_0) A dx \qquad [1]$$

To perform this integration we must obviously find a relationship between p and x. For this purpose we borrow Boyle's law:

$$pV = p_c V_c \qquad [2]$$

(Boyle's law stated in this form strictly applies to gases in static equilibrium and of constant temperature. We assume that the cavity is emptied at such slow rate that we essentially have s 'pseudo static' situation.)

Where the total gas volume V is:

$$V = V_c + Ax \qquad [3]$$

Combination of [2] and [3] gives the sought relationship between p and x.

$$p = \frac{p_c V_c}{V_c + Ax} \qquad [4]$$

The maximum piston stroke $x = x_{\max}$ is obtained by setting $p = p_0$ in Eq. [4]:

$$x_{\max} = \frac{V_c}{A} \cdot \left(\frac{p_c}{p_0} - 1 \right) \qquad [5]$$

By substituting these expressions for p and $x_{\max}$ into integral [1] it can be readily integrated.

Integration gives the simple energy stored expression:

$$W_{stored} = p_c V_c \left[\ln\left(\frac{p_c}{p_0} \right) - \left(1 - \frac{p_0}{p_c} \right) \right] \qquad [6]$$

Let us consider the following numerical case: $V_c = 10^6\ \text{m}^3$, $p_0 = 100$ kPa en $p_c = 1000$ kPa.

We get upon substitution into formula [6]: 1,40 TJ