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Secretariado IPhO 2018
Sociedade Portuguesa de Física
Avenida da República Nº45 3ºEsq
1050-187 Lisboa, Portugal

+351 21 799 36 65
info@ipho2018.pt

# Solutions to Experimental Problem 1

## Paper transistor

(Elvira Fortunato, Luís Pereira, Rui Igreja, Paul Grey, Inês Cunha, Diana Gaspar, Rodrigo Martins)

July 23, 2018

v1.4

# Sketch of the solutions:

## Part A. Circuit dimensioning (2.4 points)

## A.1

Using Ohm's law, the current through the voltage divisor is $I = V_{\text{in}}/(R_x + R_y)$, and $V_{\text{out}} = R_y I$. Thus

**A.1** 0.2pt

$$V_{\text{out}} = V_{\text{in}} \frac{R_y}{R_x + R_y}$$

## A.2

**A.2** Uncertainty in each measurement: ±0.01 Ω 0.5pt

| $\#$ | $R_{\text{T1}}$ | $R_{\text{T2}}$ | $R_{\text{T3}}$ |
|---|---|---|---|
| 1 | 122.3 | 125.3 | 125.3 |
| 2 | 122.3 | 125.4 | 125.4 |
| 3 | 122.3 | 125.3 | 125.4 |
| 4 | 122.2 | 125.2 | 125.5 |
| 5 | 122.3 | 125.4 | 125.4 |
| 6 | 122.3 | 125.4 | 125.3 |
| 7 | 122.2 | 125.4 | 125.4 |
| 8 | 122.2 | 125.3 | 125.4 |
| 9 | 122.2 | 125.4 | 125.4 |
| 10 | 122.2 | 125.4 | 125.5 |
| $\overline{R}$ | 122.25 | 125.35 | 125.40 |
| $\sigma_R$ | 0.05 | 0.07 | 0.07 |

# A.3

| | | |
|---|---|---|
| **A.3** | For a parallelepiped conductor of length $l$, width $w$ and thickness $t$, the resistance is given by $$R = \rho \frac{l}{w\,t}$$ For a thin film of square shape, $l = w$, thus $$R = \rho \frac{\not{l}}{t\not{w}} = \frac{\rho}{t} = R_{\square}.$$ | 0.3pt |

# A.4

The weighted average value (weighed by $1/\sigma^2$ ) of the sheet resistance is $\overline{R} = 123.94 \pm 0.04\,\Omega$ and $\rho = R_{\square} t$.

| | | |
|---|---|---|
| **A.4** | $\overline{R} = 123.94 \pm 0.04\,\Omega$<br>$\rho = 2.5 \pm 0.1 \times 10^{-3}\,\Omega$ m. | 0.4pt |

# A.5

| | | |
|---|---|---|
| **A.5** | For a rectangular thin film $R = R_{\square}\frac{l}{w}$, thus<br>$R_1 = R_2 = R_{\square}\left(1 + 1/0.9 + 1/0.8 + 1/0.7 + 1/0.6 + 1/0.5 + 1/0.4 + 1/0.3\right) = 14.2897 R_{\square}$<br>Measured values:<br>$R_1 = 1776 \pm 1\Omega \quad k_1 = 14.33$<br>$R_2 = 1787 \pm 1\Omega \quad k_2 = 14.42$<br>$\overline{\kappa} = 14.3 \pm 0.1$<br>Comparison with the theoretical value: the average value is compatible, within the assigned error bar, with the theoretical value. | 0.5pt |

## A.6

**A.6** Uncertainty in resistance measurements: ±1 Ω. 0.3pt

Resistor $R_1$:

| Points | $R_x/\Omega$ | $R_y/\Omega$ |
|---|---|---|
| Z | 1776 | 0 |
| A | 1708 | 165 |
| B | 1578 | 296 |
| C | 1421 | 452 |
| D | 1239 | 607 |
| E | 1033 | 829 |
| F | 768 | 1072 |
| G | 439 | 1394 |
| V | 0 | 1782 |

Resistor $R_2$:

| Points | $R_x/\Omega$ | $R_y/\Omega$ |
|---|---|---|
| Z | 1791 | 0 |
| H | 1428 | 411 |
| I | 1120 | 737 |
| J | 882 | 996 |
| K | 670 | 1200 |
| L | 498 | 1396 |
| M | 341 | 1555 |
| N | 188 | 1719 |
| W | 0 | 1793 |

## A.7

**A.7** 0.3pt

| Points | $V_{\text{out}}/\text{V}$ | Points | $V_{\text{out}}/\text{V}$ |
|---|---|---|---|
| Z | 0 | – | — |
| A | -0.208 | H | 0.664 |
| B | -0.435 | I | 1.171 |
| C | -0.699 | J | 1.593 |
| D | -1.003 | K | 1.939 |
| E | -1.337 | L | 2.24 |
| F | -1.756 | M | 2.51 |
| G | -2.29 | N | 2.77 |
| V | -2.99 | W | 3.00 |



## Part B. Characteristic Curves of the JFET transistor (4.5 points)

## B.1

**B.1** $I_{\text{DS}} = 11.84 \pm 0.01$ mA 0.2pt

## B.2

**B.2** $I_{\text{DS}}$ currents in mA: 0.8pt

| Gate/Drain | Z | H | I | J | K | L | M | N | W |
|---|---|---|---|---|---|---|---|---|---|
| Z | 0 | 1.58 | 2.18 | 2.82 | 3.60 | 4.75 | 6.45 | 9.43 | 11.87 |
| A | 0 | 1.52 | 2.13 | 2.67 | 3.47 | 4.53 | 6.04 | 7.82 | 8.78 |
| B | 0 | 1.45 | 2.00 | 2.63 | 3.29 | 4.21 | 5.15 | 5.77 | 6.09 |
| C | 0 | 1.28 | 1.79 | 2.23 | 2.59 | 2.85 | 2.99 | 3.08 | 3.16 |
| D | 0 | 0.65 | 0.76 | 0.81 | 0.85 | 0.89 | 0.92 | 0.94 | 0.96 |
| E | 0 | 0.03 | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 | 0.06 | 0.07 |
| F | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| V | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

## B.3

The unloaded voltage is

$$V_{\text{out}} = V_{\text{in}} \frac{R_y}{R_x + R_y}$$

and the loaded voltage is

$$V_{\text{out}}^{\text{L}} = V_{\text{in}} \frac{R'_y}{R_x + R'_y},$$

where $R'_y$ is the equivalent resistance of the parallel association between $R_y$ and $R_{\text{L}}$:

$$R'_y = \frac{R_y R_{\text{L}}}{R_y + R_{\text{L}}}.$$

Thus,

$$f = \frac{\frac{R'_y}{R_x+R'_y}}{\frac{R_y}{R_x+R_y}} = \frac{(R_x+R_y)R'_y}{(R_x+R'_y)R_y} = \frac{(R_x+R_y)\frac{R_{\text{L}}}{R_y+R_{\text{L}}}}{R_x + R_y \frac{R_{\text{L}}}{R_y+R_{\text{L}}}}$$

Note that in terms of $\eta = 1/(1+\frac{R_y}{R_{\text{L}}})$, the factor $f$ can be written as

$$f = \frac{(R_x+R_y)\eta}{R_x + R_y\eta}$$

When $R_\text{L} \gg R_y$, $\eta \to 1$, and $f \to 1$; when $R_\text{L} \ll R_y$, $\eta \to 0$ and $f \to 0$.

**B.3** 0.2pt

$$f = \frac{(R_x + R_y)\eta}{R_x + R_y\eta}$$

## B.4

**B.4** 0.7pt

Gate: A $V_\text{GS} = 0$ V $R_\text{DS} = 50.0$

| Drain | $V_\text{out}$/V | $V^L_\text{out}$/V | $V_\text{DS}/V$ | $I_\text{DS}$/mA | $rI$/V | $f$ |
|---|---|---|---|---|---|---|
| Z | 0,000 | 0,000 | 0,000 | 0,00 | 0,000 | 1,000 |
| H | 0,664 | 0,105 | 0,089 | 1,58 | 0,016 | 0,158 |
| I | 1,171 | 0,139 | 0,117 | 2,18 | 0,022 | 0,119 |
| J | 1,593 | 0,181 | 0,153 | 2,82 | 0,028 | 0,114 |
| K | 1,939 | 0,237 | 0,201 | 3,60 | 0,036 | 0,122 |
| L | 2,240 | 0,315 | 0,267 | 4,75 | 0,048 | 0,140 |
| M | 2,510 | 0,443 | 0,379 | 6,45 | 0,065 | 0,177 |
| N | 2,770 | 0,724 | 0,630 | 9,43 | 0,094 | 0,261 |
| W | 3,000 | 3,000 | 2,881 | 11,87 | 0,119 | 1,000 |

**B.4 cont.** 0.7pt

Gate: B $V_{\text{GS}} = -0.208$ V $R_{\text{DS}} = 58.73$

| Drain | $V_{\text{out}}$/V | $V_{\text{out}}^L$/V | $V_{\text{DS}}/V$ | $I_{\text{DS}}$/mA | $rI$/V | $f$ |
|---|---|---|---|---|---|---|
| Z | 0.000 | 0.000 | 0.000 | 0.00 | 0.000 | 1.000 |
| H | 0.664 | 0.118 | 0.102 | 1.52 | 0.015 | 0.177 |
| I | 1.171 | 0.157 | 0.136 | 2.13 | 0.021 | 0.134 |
| J | 1.593 | 0.204 | 0.177 | 2.67 | 0.027 | 0.128 |
| K | 1.939 | 0.267 | 0.233 | 3.47 | 0.035 | 0.138 |
| L | 2.240 | 0.353 | 0.308 | 4.53 | 0.045 | 0.158 |
| M | 2.510 | 0.495 | 0.435 | 6.04 | 0.060 | 0.197 |
| N | 2.770 | 0.799 | 0.721 | 7.82 | 0.078 | 0.289 |
| W | 3.000 | 3.000 | 2.912 | 8.78 | 0.088 | 1.000 |

Gate: C $V_{\text{GS}} = -0.435$ V $R_{\text{DS}} = 72.54$

| Drain | $V_{\text{out}}$/V | $V_{\text{out}}^L$/V | $V_{\text{DS}}/V$ | $I_{\text{DS}}$/mA | $rI$/V | $f$ |
|---|---|---|---|---|---|---|
| Z | 0.000 | 0.000 | 0.000 | 0.00 | 0.000 | 1.000 |
| H | 0.664 | 0.136 | 0.122 | 1.45 | 0.015 | 0.205 |
| I | 1.171 | 0.183 | 0.163 | 2.00 | 0.020 | 0.157 |
| J | 1.593 | 0.239 | 0.213 | 2.63 | 0.026 | 0.150 |
| K | 1.939 | 0.312 | 0.279 | 3.29 | 0.033 | 0.161 |
| L | 2.240 | 0.411 | 0.369 | 4.21 | 0.042 | 0.184 |
| M | 2.510 | 0.572 | 0.520 | 5.15 | 0.052 | 0.228 |
| N | 2.770 | 0.907 | 0.850 | 5.77 | 0.058 | 0.328 |
| W | 3.000 | 3.000 | 2.939 | 6.09 | 0.061 | 1.000 |

**B.4 cont.** 0.7pt

Gate: D $V_{\text{GS}} = -0.699$ V $R_{\text{DS}} = 99.86$

| Drain | $V_{\text{out}}$/V | $V^L_{\text{out}}$/V | $V_{\text{DS}}/V$ | $I_{\text{DS}}$/mA | $rI$/V | $f$ |
|---|---|---|---|---|---|---|
| Z | 0.000 | 0.000 | 0.000 | 0.00 | 0.000 | 1.000 |
| H | 0.664 | 0.170 | 0.157 | 1.28 | 0.013 | 0.256 |
| I | 1.171 | 0.232 | 0.214 | 1.79 | 0.018 | 0.198 |
| J | 1.593 | 0.303 | 0.281 | 2.23 | 0.022 | 0.190 |
| K | 1.939 | 0.395 | 0.369 | 2.59 | 0.026 | 0.204 |
| L | 2.240 | 0.516 | 0.487 | 2.85 | 0.029 | 0.230 |
| M | 2.510 | 0.708 | 0.678 | 2.99 | 0.030 | 0.282 |
| N | 2.770 | 1.089 | 1.059 | 3.08 | 0.031 | 0.393 |
| W | 3.000 | 3.000 | 2.968 | 3.16 | 0.032 | 1.000 |

Gate: E $V_{\text{GS}} = -1.003$ V $R_{\text{DS}} = 176.3$

| Drain | $V_{\text{out}}$/V | $V^L_{\text{out}}$/V | $V_{\text{DS}}/V$ | $I_{\text{DS}}$/mA | $rI$/V | $f$ |
|---|---|---|---|---|---|---|
| Z | 0.000 | 0.000 | 0.000 | 0.00 | 0.000 | 1.000 |
| H | 0.664 | 0.245 | 0.238 | 0.65 | 0.007 | 0.369 |
| I | 1.171 | 0.346 | 0.338 | 0.76 | 0.008 | 0.295 |
| J | 1.593 | 0.454 | 0.446 | 0.81 | 0.008 | 0.285 |
| K | 1.939 | 0.586 | 0.578 | 0.85 | 0.009 | 0.302 |
| L | 2.240 | 0.754 | 0.745 | 0.89 | 0.009 | 0.337 |
| M | 2.510 | 1.004 | 0.994 | 0.92 | 0.009 | 0.400 |
| N | 2.770 | 1.451 | 1.441 | 0.94 | 0.009 | 0.524 |
| W | 3.000 | 3.000 | 2.990 | 0.96 | 0.010 | 1.000 |

**B.4 cont.** 1.2pt

Gate: F $V_{\text{GS}} = -1.337$ V $R_{\text{DS}} = 1111$

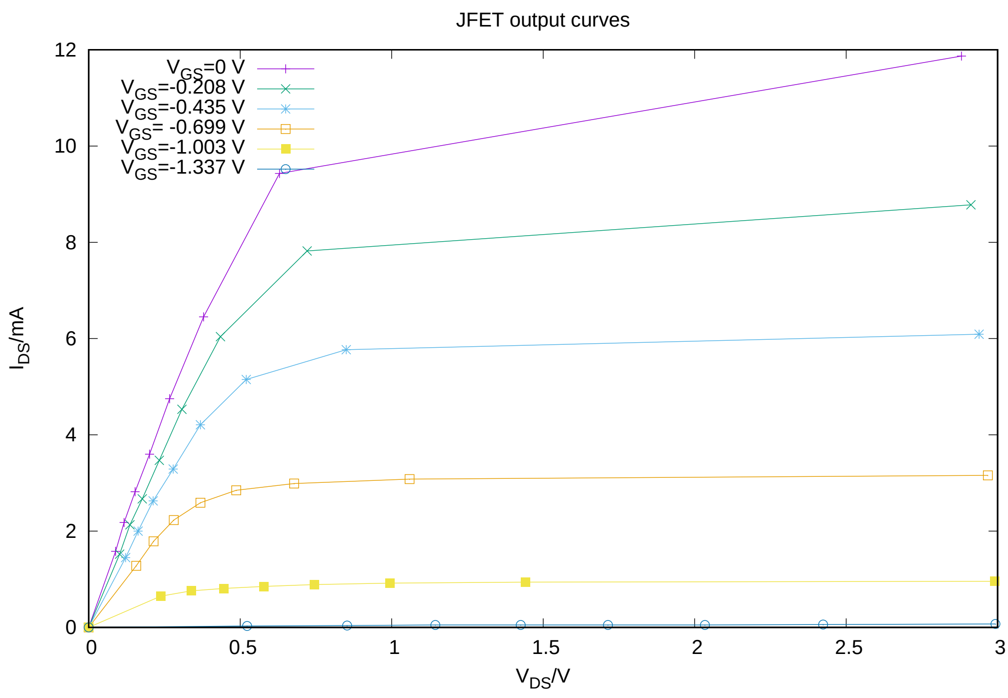
| Drain | $V_{\text{out}}/\text{V}$ | $V^L_{\text{out}}/\text{V}$ | $V_{\text{DS}}/V$ | $I_{\text{DS}}/\text{mA}$ | $rI/\text{V}$ | $f$ |
|---|---|---|---|---|---|---|
| Z | 0.000 | 0.000 | 0.000 | 0.00 | 0.000 | 1.000 |
| H | 0.664 | 0.526 | 0.523 | 0.03 | 0.003 | 0.791 |
| I | 1.171 | 0.857 | 0.853 | 0.04 | 0.004 | 0.732 |
| J | 1.593 | 1.149 | 1.144 | 0.05 | 0.005 | 0.721 |
| K | 1.939 | 1.431 | 1.426 | 0.05 | 0.005 | 0.738 |
| L | 2.240 | 1.719 | 1.714 | 0.05 | 0.005 | 0.767 |
| M | 2.510 | 2.039 | 2.034 | 0.05 | 0.005 | 0.812 |
| N | 2.770 | 2.430 | 2.424 | 0.06 | 0.006 | 0.877 |
| W | 3.000 | 3.000 | 2.993 | 0.07 | 0.007 | 1.000 |

Gate: G $V_{\text{GS}} = -1.756$ V $R_{\text{DS}} = \infty$

| Drain | $V_{\text{out}}/\text{V}$ | $V^L_{\text{out}}/\text{V}$ | $V_{\text{DS}}/V$ | $I_{\text{DS}}/\text{mA}$ | $rI/\text{V}$ | $f$ |
|---|---|---|---|---|---|---|
| Z | 0.000 | 0.000 | 0.000 | 0.00 | 0.000 | 1.000 |
| H | 0.664 | -0.288 | -0.288 | 0.00 | 0.000 | -0.434 |
| I | 1.171 | -0.325 | -0.325 | 0.00 | 0.000 | -0.278 |
| J | 1.593 | -0.415 | -0.415 | 0.00 | 0.000 | -0.260 |
| K | 1.939 | -0.562 | -0.562 | 0.00 | 0.000 | -0.290 |
| L | 2.240 | -0.800 | -0.800 | 0.00 | 0.000 | -0.357 |
| M | 2.510 | -1.325 | -1.325 | 0.00 | 0.000 | -0.528 |
| N | 2.770 | -3.675 | -3.675 | 0.00 | 0.000 | -1.327 |
| W | 3.000 | 3.000 | 3.000 | 0.00 | 0.000 | 1.000 |

## B.5

**B.5** Output curves: 0.5pt




## B.6

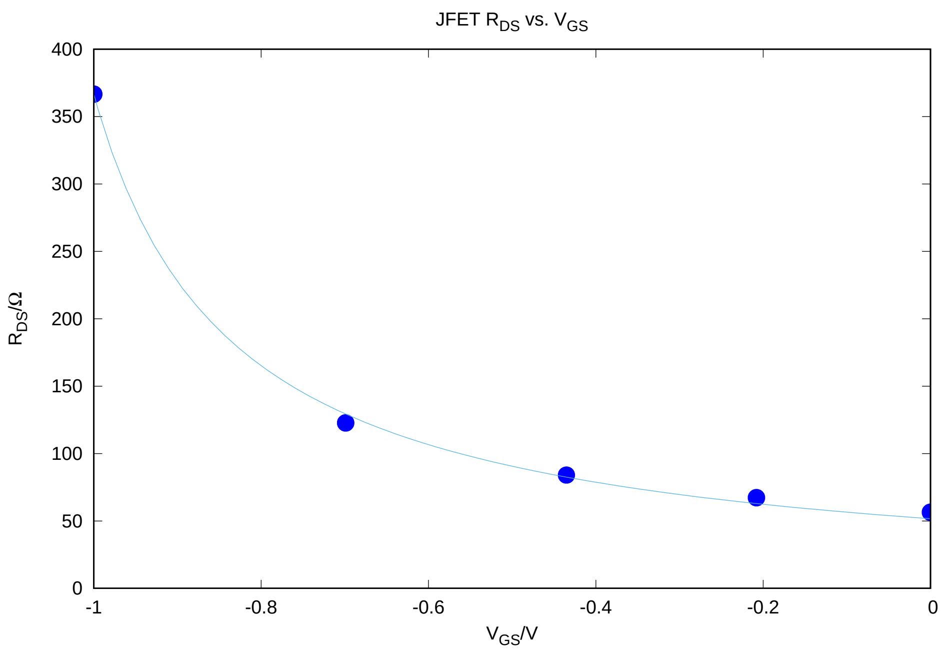
The $R_{\text{DS}}$ values are obtained from the slopes of the linear region of the output curves (small $V_{\text{DS}}$ voltages).

The last point in the plot $R_{\text{DS}}(V_{\text{GS}})$ has a large error bars as we are missing points in the linear regime, and will be ignored.

The solid line in the plot is the result of a fit to $R_{\text{DS}} = R^0_{\text{DS}}\,(1 - V_{\text{GS}}/V_{\text{P}})$, that gave $R^0_{\text{DS}} = 52(2)\ \Omega$, $V_{\text{P}} = -1.18(1)$ V.

**B.6** 0.5pt

| $V_{GS}$/V | $R_{DS}/\Omega$ |
|---|---|
| 0 | 56.5 ±2 |
| -0.208 | 67.4 ±2 |
| -0.435 | 84.1 ±4 |
| -0.699 | 122.84 ±4 |
| -1.003 | 366.6 ±4 |
| -1.337 | 1111 ±100 |



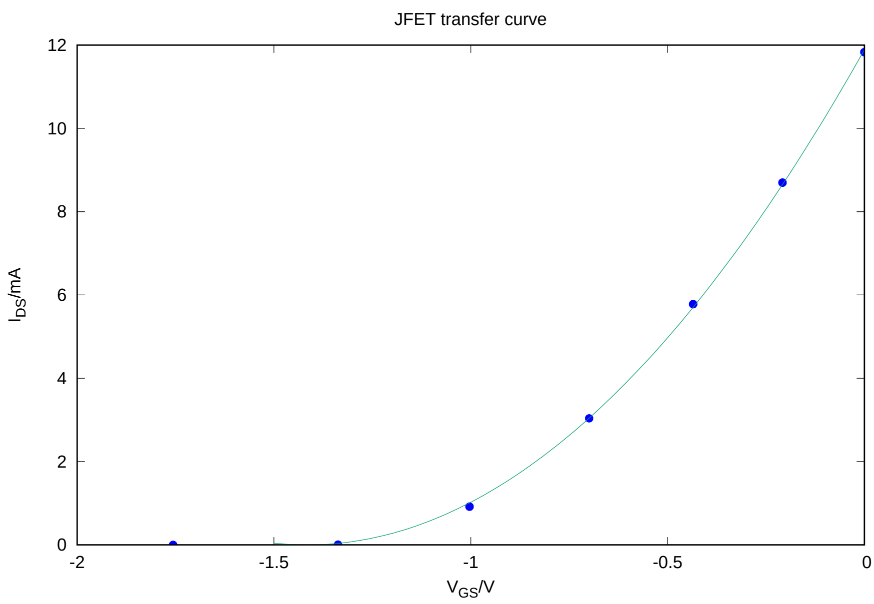
## B.7

The data was obtained with $V_{\mathrm{DS}} = +3$ V. The solid line is the result of the fit to the data of the function

$$I_{\mathrm{DS}} = I_{\mathrm{DSS}}\left(1 - V_{\mathrm{GS}}/V_{\mathrm{P}}\right)^2 .$$

The fitted parameters are $I_{\mathrm{DSS}} = 11.89 \pm 0.06$ mA and $V_{\mathrm{P}} = -1.42 \pm 0.02$ V.

**B.7** 0.3pt

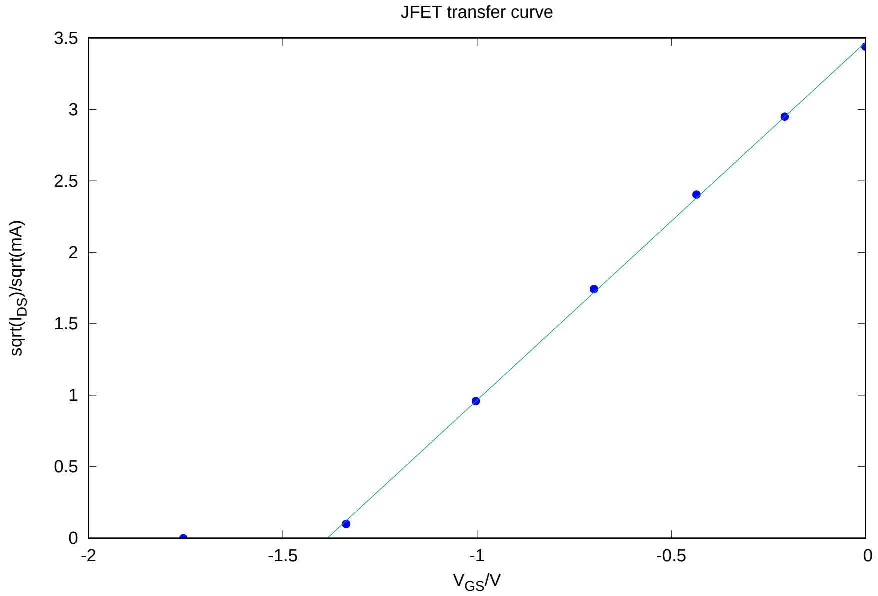



## B.8

From

$$I_{\mathrm{DS}} = I_{\mathrm{DSS}}\left(1 - V_{\mathrm{GS}}/V_{\mathrm{P}}\right)^2$$

a plot of $\sqrt{I_{\mathrm{DS}}}$ as function of $V_{\mathrm{GS}}$ should yield a straight line with slope $a = -\sqrt{I_{\mathrm{DS}}}/V_{\mathrm{P}}$ that intercepts the $x-$axis at $V_{\mathrm{P}}$.

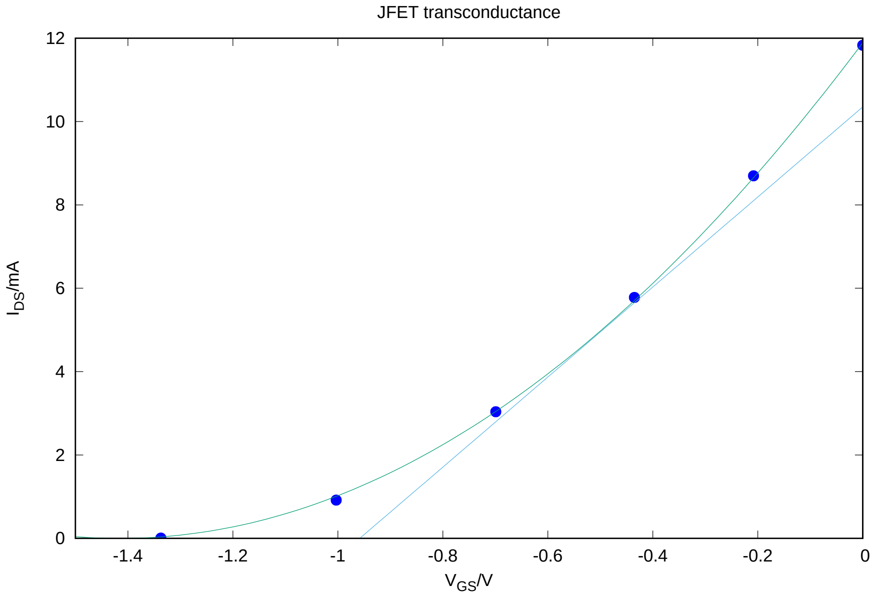



A linear fit to $fx) = a\,x + b$ gave $a = 2.50(2)$ and $b = 3.47(2)$. Thus, $V_{\text{P}} = -b/a = -1.39(2)$ V and $I_{\text{DSS}} = 4.23^2 = 12.0(2)$ mA.

| | | |
|---|---|---|
| **B.8** | $V_{\text{P}} = -b/a = -1.39(2)$ V<br>$I_{\text{DSS}} = 4.23^2 = 12.0(2)$ mA. | 0.4pt |

## B.9

The transcondutance is the slope of the transfer curve at a given point. From the transfer plot, we draw the tangent at the point with abscissa $-0.50$ V and read the slope from the graph, obtaining $g = 10.8(1)$ m $^{-1}$.




From

$$I_{\mathsf{D}} = I_{\mathsf{DSS}} \left(1 - V_{\mathsf{GS}}/V_{\mathsf{P}}\right)^2 ,$$

$$g = \frac{\partial I_{\mathsf{DS}}}{\partial V_{\mathsf{GS}}} = 2 I_{\mathsf{DSS}} \left(1 - V_{\mathsf{GS}}/V_{\mathsf{P}}\right) \left(-\frac{1}{V_{\mathsf{P}}}\right) = \frac{2 I_{\mathsf{DSS}}}{V_{\mathsf{P}}} \left(V_{\mathsf{GS}}/V_{\mathsf{P}} - 1\right) .$$

Substituting values,

$$g = 10.8\ \mathsf{m}^{-1}$$

a value that agrees with that obtained using the graphical method.

| | | |
|---|---|---|
| **B.9** | $g_{\mathsf{measured}} = 10.8(1)\ \mathsf{m}^{-1}$<br>$g_{\mathsf{model}} = 10.8\ \mathsf{m}^{-1}$ | 0.4pt |

## Part C: The Paper Thin Film Transistor (2.0 points)

## C.1

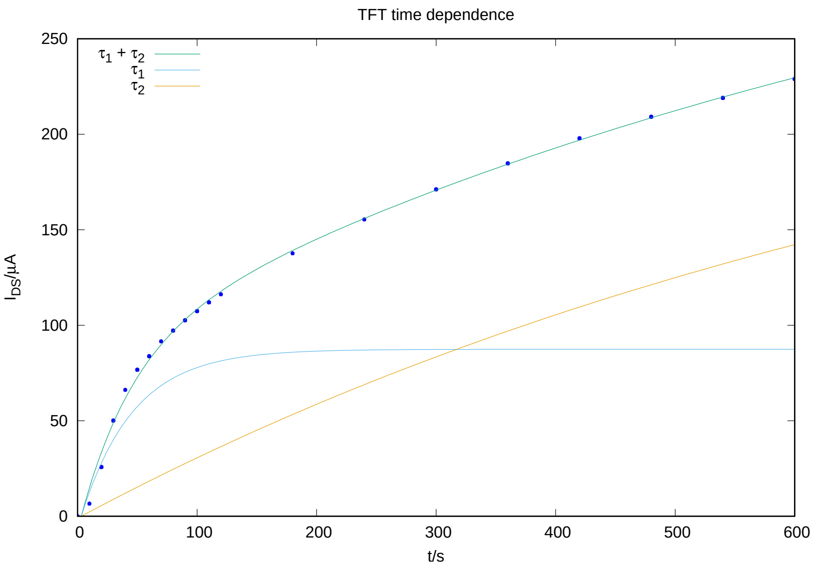
**C.1** 0.8pt

| $t$/s | $I_{DS}/\mu$A | $t$/s | $I_{DS}/\mu$A |
|---|---|---|---|
| 0 | 0 | 110 | 112,0 |
| 10 | 6.6 | 120 | 116.2 |
| 20 | 25.8 | 180 | 137.7 |
| 30 | 50.1 | 240 | 155.4 |
| 40 | 66.2 | 300 | 171.2 |
| 50 | 76.7 | 360 | 184.4 |
| 60 | 83.8 | 420 | 197.9 |
| 70 | 91.6 | 480 | 209.2 |
| 80 | 97.2 | 540 | 219.1 |
| 90 | 102.6 | 600 | 220.0 |
| 100 | 107.4 | - | - |

## C.2

The data is similar to that of the charge of a capacitor, superimposed with an almost linear component that corresponds to the charge of the second capacitor with a larger time constant.

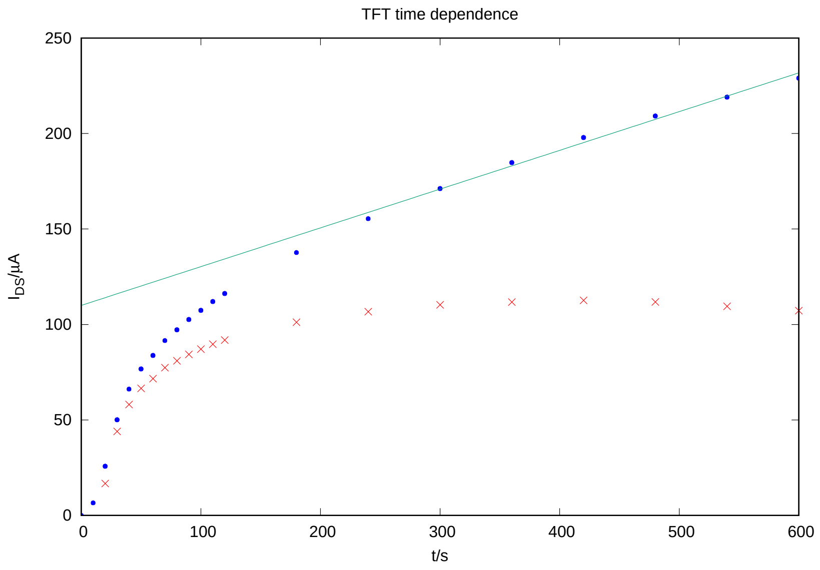
A least squares fit to a $A(1-\exp(-t/\tau_1)) + B(1-\exp(-t/\tau_2))$ is also depicted, showing that the data can be well fitted by this model. The shorter time constant is $\tau_1 = 43(8)$ s, the longer time constant, $\tau_2$ is roughly 20 times larger.

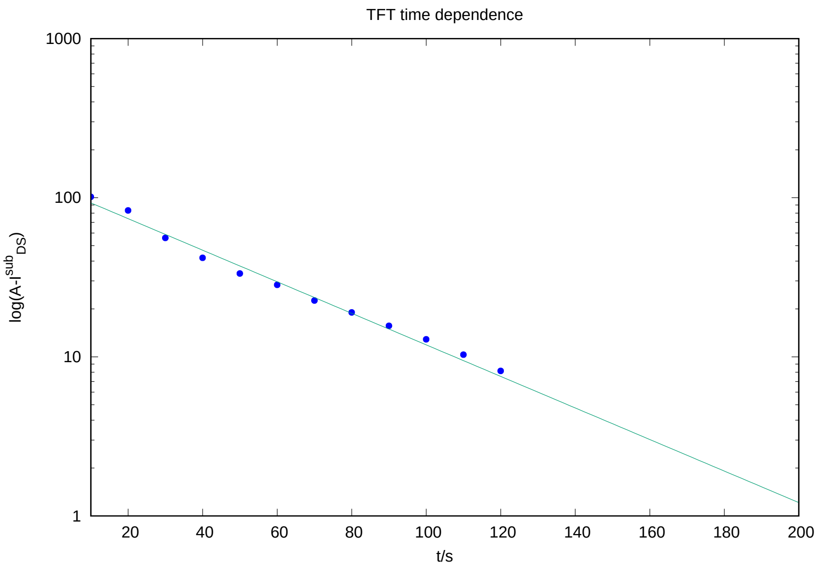



Let $I_{\text{DS}}^{\text{sub}} = A(1-\exp(-t/\tau_1))$ be the data subtracted from the long time constant component. A logarithmic plot of $\log(A - I_{\text{DS}}^{\text{sub}})$ should be a straight line of slope $-1/\tau_1$. The constant $A$, the saturation current of the short $\tau_1$ component, can be easily estimated from the above plot.

The slope of the line is $m = -0.023(1)$, from which we get $\tau_1 = 44(3)$ s. The error bar is underestimated, as it does not take into account the error in the subtraction of the $\tau_2$ component.

**C.2** 1.2pt




$\tau_1 = 44(3)$ s.

## Part D. Inverter Circuit (1.0 points)

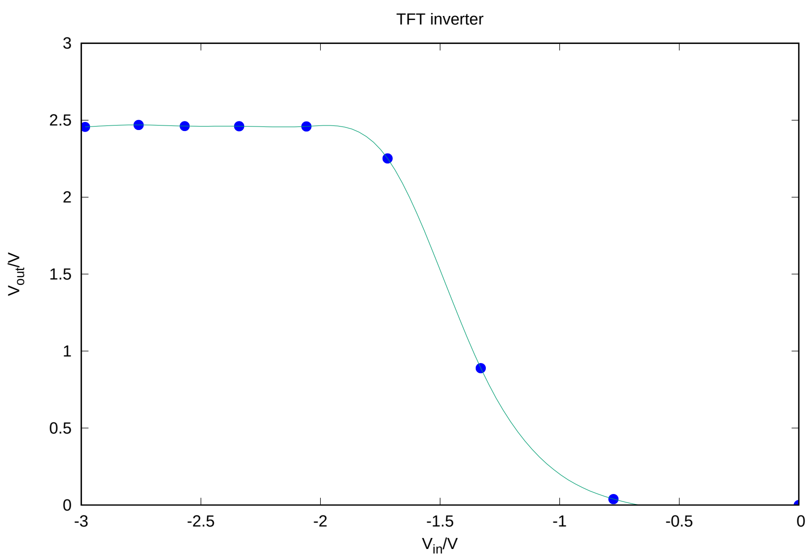
## D.1

**D.1** $R_L = 198$ kΩ 0.5pt

| $t$ | $V_{in}$/V | $V_{out}$/V |
|---|---|---|
| | -2.983 | 2.456 |
| | -2.760 | 2.470 |
| | -2.567 | 2.461 |
| | -2.340 | 2.461 |
| | -2.058 | 2.460 |
| | -1.719 | 2.252 |
| | -1.330 | 0.889 |
| | -0.775 | 0.039 |

## D.2

**D.2** 0.5pt



IPhO 2018
Lisbon, Portugal

Secretariado IPhO 2018
Sociedade Portuguesa de Física
Avenida da República Nº45 3ºEsq
1050-187 Lisboa, Portugal

+351 21 799 36 65
info@ipho2018.pt

# Detailed Marking Scheme Experimental Problem 1

## Paper transistor

(Elvira Fortunato, Luís Pereira, Rui Igreja, Paul Grey, Inês Cunha, Diana Gaspar, Rodrigo Martins)

July 22, 2018

v1.4

Secretariado
IPhO 2018
Sociedade Portuguesa de Física
Avenida da República nº45 3ºEsq
1050-187 Lisboa, Portugal

# Paper transistor (10 points)

## Part A. Circuit dimensioning (2.5 points)

### A.1

| | |
|---|---|
| Apply Ohm's Law | 0.1 |
| Obtain the output voltage | 0.1 |
| Total | 0.2 |

### A.2

| | |
|---|---|
| Five or more measurement for each resistance | 0.3 |
| Calculate the average | 0.1 |
| Uncertainty | 0.1 |
| Total | 0.5 |

[not reasonable enough number of points, $-0.2$pt]

### A.3

| | |
|---|---|
| Expression for the resistance | 0.1 |
| Obtain $R_{\square}$ | 0.2 |
| Total | 0.3 |

### A.4

| | |
|---|---|
| Calculate the weighted average sheet resistance | 0.2 |
| Obtain the resistivity | 0.2 |
| Total | 0.4 |

[missing the uncertainty, $-0.1$pt]

### A.5

| | |
|---|---|
| Expression for the theoretical $\kappa$ | 0.2 |
| Measurement of the resistances | 0.1 |
| Experimental value of $\kappa$ and comparison | 0.2 |
| Total | 0.5 |

## A.6

| | |
|---|---|
| Measurement of the resistances $R_x$ and $R_y$ | 0.3 |
| Total | 0.3 |

[missing units in the table, $-0.05$pt; mixing up $R_x$ and $R_y$, $-0.1$pt]

## A.7

| | |
|---|---|
| Measurement of all $V_{\text{out}}$ values | 0.3 |
| Total | 0.3 |

[missing or wrong unit, $-0.05$pt; wrong sign of $V_{\text{out}}$, $-0.1$pt]

**Part B. Characteristic Curves of the JFET transistor (4.5 points)**

## B.1

| | |
|---|---|
| Value within 20% of the correct value | 0.2 |
| Total | 0.2 |

[missing or wrong unit $-0.05$pt; missing uncertainty $-0.05$pt]

## B.2

| | |
|---|---|
| Measurements of $I_{\text{DS}}$ (first part) | 0.3 |
| Measurements of $I_{\text{DS}}$ (second part, at least four sets of measurements for $V_{\text{GS}} < 0$) | 0.4 |
| Five or more sets of measurements for $V_{\text{GS}} < 0$ | 0.1 |
| Total | 0.8 |

[wrong or missing current units, $-0.1$pt; wrong number of significant digits in table entries, $-0.05$pt]

## B.3

| | |
|---|---|
| Expression for $f$ | 0.2 |
| Total | 0.2 |

## B.4

| | |
|---|---|
| Realize that $R_{\text{L}} = R_{\text{DS}} +$ internal resistance of multimeter | 0.2 |
| Calculation of $R_{\text{DS}}$ from nominal data | 0.3 |
| Apply correction factor $f$ | 0.5 |
| Subtract the voltage drop inside the multimeter | 0.2 |
| Total | 1.2 |

## B.5

| | |
|---|---|
| Plot, at least, five curves (0.1pt each) | 0.5 |
| Total | 0.5 |

[use uncorrected $V_{\text{DS}}$, $-0.1$ pt; wrong or missing axes labels, $-0.1$pt]

## B.6

| | |
|---|---|
| Obtain the experimental values from slopes | 0.3 |
| Plot the graph | 0.2 |
| Total | 0.5 |

[any reasonable graph is worth 0.2; no graph analysis required]

## B.7

| | |
|---|---|
| Draw a good plot | 0.3 |
| Total | 0.3 |

[wrong or missing magnitudes in axes labels, $-0.05$pt; wrong or missing units in axes labels, $-0.05$pt; plot the curve for a wrong $V_{\text{DS}}$, $-0.3$pt; graph showing unreasonable deviation with respect to the transistor data, $-0.2$pt]

## B.8

| | |
|---|---|
| Current $I_{\text{DSS}}$ | 0.1 |
| Obtain $V_{\text{p}}$ using the appropriate graphical method (plotting $\sqrt{I_{\text{DS}}}$ as a function of $V_{\text{GS}}$) | 0.2 |
| Comparison | 0.1 |
| Total | 0.4 |

[if $V_{\text{p}}$ is not obtained from an appropriate graphical method, using a plot with linearized data, -$0.05$pt]

### B.9

| | |
|---|---|
| Plot the transconductance curve | 0.1 |
| Obtain $g$ from the slope of the tangent to the curve | 0.2 |
| Comparison with model equation (2) | 0.1 |
| Total | 0.4 |

## Part C: The Paper Thin Film Transistor (2.0 points)

### C.1

| | |
|---|---|
| At least fifteen data points presented in the table | 0.8 |
| Total | 0.8 |

[not using the appropriate multimeter range (2000 $\mu$A), $-0.1$pt; missing units, $-0.1$pt; data deviating too much from the expected behaviour, $-0.2$pt; IDS for the closed transistor is not the 1st value, $-0.1$pt; not enough points in the fast changing regime, $-0.1$pt]

### C.2

| | |
|---|---|
| Draw a good plot | 0.3 |
| Show the similarity with two parallel $RC$ circuits | 0.1 |
| Subtraction of the long-time constant component | 0.4 |
| Determine $\tau_1$ | 0.4 |
| Total | 1.2 |

[missing or wrong units in the plot $-0.1$pt]

## Part D. Inverter Circuit (1.0 points)

### D.1

| | |
|---|---|
| Measurement of $R_\mathrm{L}$ in the correct range | 0.1 |
| Measurement of, at least, eight points | 0.2 |
| Data showing a clear cut-off ($V_\mathrm{out}$ should go below 0.1 V) | 0.2 |
| Total | 0.5 |

[missing units and/or wrong labels, $-0.05$pt each; $\tau_1$ not used as time in between measurements, $-0.1$pt]

### D.2

| | |
|---|---|
| Draw a good plot, including the smooth trend curve | 0.5 |
| Total | 0.5 |

[missing units and/or wrong labels, $-0.05$pt each; non-smooth curve (e.g. trend curve with spikes), $-0.2$pt]

IPhO 2018
Lisbon, Portugal

Secretariado IPhO 2018
Sociedade Portuguesa de Física
Avenida da República Nº45 3ºEsq
1050-187 Lisboa, Portugal

# Solutions to Experimental Problem 2

+351 21 799 36 65
info@ipho2018.pt

## Viscoelasticity of a polymer thread

(J. M. Gil, J. Pinto da Cunha, R. C. Vilão, H. V. Alberto)

July 22, 2018

v1.1

# Problem 2: Viscoelasticity of a polymer thread (10 points)

## Part A. Stress-relaxation measurements (1.9 points)

### A.1

Measurement: $\ell_0 = 42.7 + 2 \times 0.5 = 43.7\,$cm ,

| A.1 | | 0.3pt |
|---|---|---|
| | $\ell_0 = (43.7 \pm 0.2)$ cm . | |

### A.2

| A.2 | | 0.3pt |
|---|---|---|
| | $P_0 = (81.11 \pm 0.03)$ gf . | |

### A.3

The table contains the readings on the scale $P$ (Question A.3) and the force on the thread, $F(t)$, at constant strain (Question D.1). The values of $\frac{dF}{dt}$ (Question D.6) were computed numerically using equal time intervals. The function $y(t)$ is given by $y(t) = F(t) - F_0 - F_1\,e^{-t/\tau_1}$ (Question D.10).

**A.3** (1.0pt)

| $t$ /s | $P(t)$ /gf | $F$ /gf | $\frac{dF}{dt}$ /gf s$^{-1}$ | $y(t)$ /gf |
|---|---|---|---|---|
| 10 | 35.7 | 45.41 | | 2.82 |
| 17 | 36.2 | 44.91 | | 2.33 |
| 26 | 36.6 | 44.51 | | 1.95 |
| 32 | 36.8 | 44.31 | | 1.76 |
| 40 | 37.0 | 44.11 | | 1.57 |
| 46 | 37.1 | 44.01 | | 1.48 |
| 51 | 37.2 | 43.91 | | 1.38 |
| 58 | 37.3 | 43.81 | | 1.29 |
| 65 | 37.4 | 43.71 | | 1.20 |

**A.3** 0.3pt

| $t$ /s | $P(t)$ /gf | $F$ /gf | $\frac{dF}{dt}$ /gf s$^{-1}$ | $y(t)$ /gf |
|---|---|---|---|---|
| 73 | 37.5 | 43.61 | | 1.12 |
| 84 | 37.6 | 43.51 | | 1.03 |
| 94 | 37.7 | 43.41 | | 0.94 |
| 105 | 37.8 | 43.31 | | 0.86 |
| 118 | 37.9 | 43.21 | | 0.77 |
| 136 | 38.0 | 43.11 | | 0.70 |
| 151 | 38.1 | 43.01 | | 0.62 |
| 173 | 38.2 | 42.91 | | 0.55 |
| 193 | 38.3 | 42.81 | | 0.48 |
| 217 | 38.4 | 42.71 | | 0.41 |
| 247 | 38.5 | 42.61 | | 0.35 |
| 279 | 38.6 | 42.51 | | 0.29 |
| 317 | 38.7 | 42.41 | | 0.23 |
| 358 | 38.8 | 42.31 | | 0.18 |
| 408 | 38.9 | 42.21 | | 0.14 |
| 471 | 39.0 | 42.11 | | 0.11 |
| 525 | 39.1 | 42.01 | | 0.07 |
| 591 | 39.2 | 41.91 | | 0.03 |
| 600 | 39.2 | 41.91 | | 0.04 |
| 672 | 39.3 | 41.81 | | 0.01 |
| 773 | 39.4 | 41.71 | | 0.007 |
| 866 | 39.5 | 41.61 | | −0.01 |
| 900 | 39.52 | 41.59 | | −0.00 |
| 993 | 39.6 | 41.51 | | −0.01 |
| 1124 | 39.7 | 41.41 | | |
| 1200 | 39.74 | 41.37 | $-7.00 \times 10^{-4}$ | |
| 1272 | 39.8 | 41.31 | | |
| 1419 | 39.9 | 41.21 | | |
| 1500 | 39.94 | 41.17 | $-5.33 \times 10^{-4}$ | |
| 1628 | 40.0 | 41.11 | | |
| 1800 | 40.06 | 41.05 | $-4.67 \times 10^{-4}$ | |
| 1869 | 40.1 | 41.01 | | |
| 2037 | 40.2 | 40.91 | | |
| 2100 | 40.22 | 40.89 | $-3.83 \times 10^{-4}$ | |
| 2400 | 40.29 | 40.82 | | |

## A.4

Measurement: $\ell = 50.0 + 2 \times 0.5 = 51.0\,\text{cm}$ ,

**A.4** 0.3pt

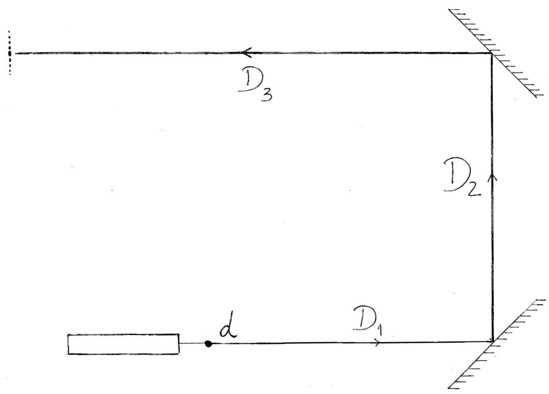
$$\ell = (51.0 \pm 0.2)\,\text{cm} .$$

# Part B. Measurement of the streched thread diameter (1.5 points)

## B.1

Two mirrors are used to maximize the distance D and consequently the distance between diffraction minima.

**B.1** Sketch of the method 0.6pt




## B.2

The total distance $D$ is the sum

$$D = D_1 + D_2 + D_3 = (26.0 + 36.0 + 102.3)\,\text{cm} = 164.3\,\text{cm} = 1.643\,\text{m} .$$

The estimated uncertainties are

$$\sigma_{D_1} = \sigma_{D_2} = \sigma_{D_3} \approx 0.5\,\text{cm} \;\Rightarrow\; \sigma_D = \sqrt{3 \times \sigma_{D_1}^2} = 0.5 \times \sqrt{3} = 0.87\,\text{cm} .$$

**B.2** 0.3pt

$$D = (1.643 \pm 0.009)\,\text{m} .$$

## B.3

The distance between minima, $x$, is quite small. To reduce the error, the total distance $Nx$, with $N = 22$, was measured:

$$22\,x = 49\,\text{mm} \;\Rightarrow\; \bar{x} = 2.227\,\text{mm}\,.$$

The corresponding uncertainty is

$$\sigma_{\bar{x}} = \frac{\sigma_{22\,x}}{22} = \frac{0.25\,\text{mm}}{22} = 0.011\,\text{mm}\,.$$

**B.3** 0.3pt

$$\bar{x} \;= (2.227 \pm 0.011)\,\text{mm}\,.$$

## B.4

Using previous results, we get

$$d = \frac{\lambda}{\sin\theta} \simeq \frac{\lambda\,D}{\bar{x}} = \frac{650\times10^{-9}\,\text{m}\times 1.643\,\text{m}}{2.227\times10^{-3}\,\text{m}} = 4.795\times10^{-4}\,\text{m} = 0.480\,\text{mm}\,.$$

For the uncertainties, we have

$$\frac{\sigma_d}{d} = \frac{\sigma_\lambda}{\lambda} + \frac{\sigma_D}{D} + \frac{\sigma_{\bar{x}}}{\bar{x}} = \frac{10}{650} + \frac{0.0087}{1.643} + \frac{0.011}{2.227} = 0.02517 \;\Rightarrow\; \sigma_d = 0.02517\times 0.480\,\text{mm} = 0.012\,\text{mm}\,.$$

**B.4** 0.3pt

$$d \;= (0.480 \pm 0.012)\,\text{mm}\,.$$

## Part C. Change to a new thread (0.3 points)

## C.1

Measurement: $\ell_0' = 31.6 + 2\times 0.5 = 32.6\,\text{cm}$.

**C.1** 0.3pt

$$\ell_0' \;= (32.6 \pm 0.2)\,\text{cm}\,.$$

## Part D. Data Analysis (5.7 points)

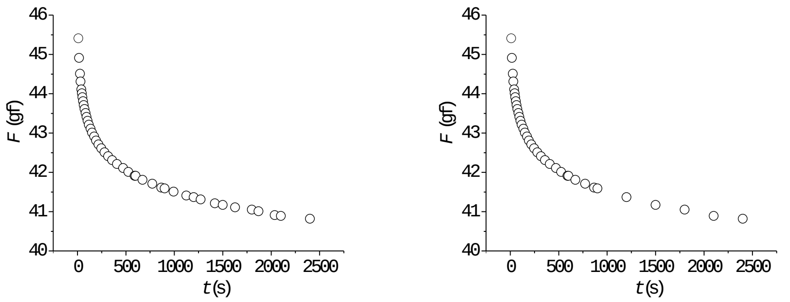
### D.1

The force on the thread was calculated as $F(t) = (P_0 - P(t))$, in gram-force units.

| | | |
|---|---|---|
| **D.1** | See column $F(t)$ in the table in A.3. | 0.3pt |

### D.2

**D.2**




**Left**: $F(t)$ sampled at unequal time intervals. **Right**: $F(t)$ sampled at equal time intervals for $t > 1000$ s.

0.4pt

### D.3

The dimensionless quantity $\epsilon$ is given by

$$\epsilon = \frac{\ell - \ell_0}{\ell_0} = \frac{51.0 - 43.7}{43.7} = 0.167 \ .$$

The uncertainty in $\epsilon$, $\sigma_\epsilon$, is calculated propagating the uncertainties in the measured length, $\sigma_\ell$ and $\sigma_{\ell_0}$:

$$\begin{aligned} \frac{\sigma_\epsilon}{\epsilon} &= \frac{\sigma_{(\ell-\ell_0)}}{\ell - \ell_0} + \frac{\sigma_{\ell_0}}{\ell_0} \\ &= \frac{\sqrt{\sigma_\ell^2 + \sigma_{\ell_0}^2}}{\ell - \ell_0} + \frac{\sigma_{\ell_0}}{\ell_0} \\ &= \frac{0.2 \times \sqrt{2}}{7.3} + \frac{0.2}{43.7} \\ &= 0.0433 \end{aligned}$$

Therefore, $\sigma_\epsilon = 0.0433 \times 0.167 = 0.0072$.

| | | |
|---|---|---|
| **D.3** | $$\epsilon = 0.167 \pm 0.007 \ .$$ | 0.3pt |

## D.4

One has

$$\frac{\sigma}{\epsilon} = \frac{F}{\epsilon\, S}\ .$$

In this case, $S = \pi(d/2)^2 = 1.809 \times 10^{-7}\,\mathrm{m}^2$ and $\epsilon = 0.167$. We also have $1\,\mathrm{gf} = g \times 10^{-3}\,\mathrm{N}$ with $g = 9.8\,\mathrm{m\,s^{-2}}$. Therefore, if $F$ is in gram-force units we have

$$\frac{\sigma}{\epsilon} = \frac{9.8 \times 10^{-3}\,\mathrm{gf}^{-1}\,\mathrm{N}}{0.167 \times 1.809 \times 10^{-7}\,\mathrm{m}^2}\ F = \left(324293\,\mathrm{gf}^{-1}\,\mathrm{N\,m^{-2}}\right)\ F\ ,$$

where $F$ is in gf , and $\sigma$ is in $\mathrm{N\,m^{-2}}$. Comparing with $\frac{\sigma}{\epsilon} = \beta\, F$ we get

$$\beta = 324293\,\mathrm{gf}^{-1}\,\mathrm{N\,m^{-2}}\ \ .$$

Note that, if we write

$$F(t) = F_0 + F_1\,\mathrm{e}^{-t/\tau_1} + F_2\,\mathrm{e}^{-t/\tau_2} + F_3\,\mathrm{e}^{-t/\tau_3} + \cdots \quad (1)$$

and compare with equation

$$\frac{\sigma}{\epsilon} = \beta\, F(t) = E_0 + E_1\,\mathrm{e}^{-t/\tau_1} + E_2\,\mathrm{e}^{-t/\tau_2} + E_3\,\mathrm{e}^{-t/\tau_3} + \cdots \quad (2)$$

we conclude that $E_0 = \beta F_0$ , $E_1 = \beta F_1$, $E_2 = \beta F_2$, etc.

**D.4** 0.3pt

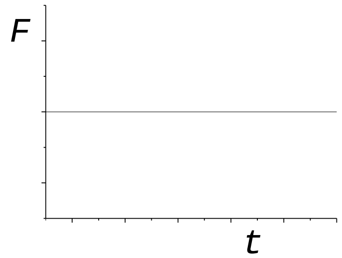
$$\beta = 3.24 \times 10^5\,\mathrm{gf}^{-1}\,\mathrm{N\,m^{-2}}\ \ .$$

## D.5

For a purely elastic process, $\sigma = \epsilon\, E_0$ and

$$F = \alpha\,\sigma = \alpha\,\epsilon\, E_0\ \ .$$

Thus, a graph of a constant function is expected.




**D.5** 0.4pt

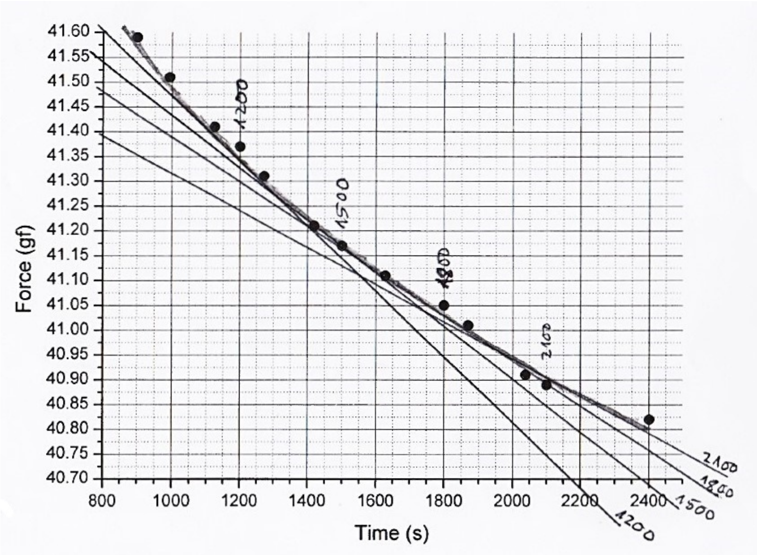
## D.6

The data for $\frac{\mathrm{d}F}{\mathrm{d}t}$ inserted in table introduced in A.3, was computed numerically for equal time intervals.

However, the graphical method is also exemplified. In the present graph, tangent lines to $F(t)$ are drawn at four different time instants (1200,1500,1800 and 2100 s). The slopes of those lines are a measure of $\frac{\mathrm{d}F}{\mathrm{d}t}$ at those instants.

**D.6** See in the table used in A.3, the column with $\frac{\mathrm{d}F}{\mathrm{d}t}$. 0.5pt

This graph is present only if a graphical method is used.




## D.7

For a single viscoelastic process,

$$F = \frac{1}{\beta}\left(E_0 + E_1\,\mathrm{e}^{-t/\tau_1}\right) = F_0 + F_1\,\mathrm{e}^{-t/\tau_1}\ .$$

Therefore,

**D.7** 0.3pt

$$\frac{\mathrm{d}F}{\mathrm{d}t} = -\frac{F_1}{\tau_1}\,\mathrm{e}^{-t/\tau_1}\ , \quad \text{where} \quad F_1 = \frac{E_1}{\beta}\ .$$

## D.8

The linearisation of the expression of $\mathrm{d}F/\mathrm{d}t$ is accomplished using logarithms:

$$\ln\left(-\frac{\mathrm{d}F}{\mathrm{d}t}\right) = \ln\left(\frac{F_1}{\tau_1}\right) - \frac{1}{\tau_1}\,t\ .$$

The plot of ln$(-\mathrm{d}F/\mathrm{d}t)$ is shown in the graph below for a case where the derivative was obtained numerically (left) and using a graphic method (right).

For the left graph, the best straight line is ln$(-\mathrm{d}F/\mathrm{d}t) = m_1\,t + b_1$ where $m_1 = (-6.47 \pm 0.62) \times 10^{-4}$ and $b_1 = (-6.52 \pm 0.11)$, using $t$ in seconds and the force in gram-force units. If the derivative is computed numerically for unequal time intervals, the final parameters $E_1$ and $\tau_1$ are similar.

The best straight line for the right graph yields $m_1 = (-6.00 \pm 0.15) \times 10^{-4}$ and $b_1 = (-6.63 \pm 0.02)$ using $t$ in seconds and the force in gram-force units.

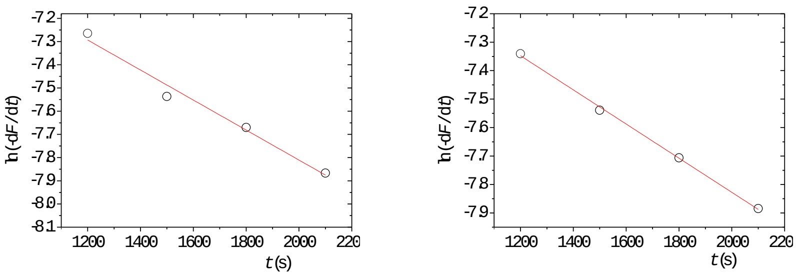
Thus, using the data from the left graph, $\tau_1 = \frac{1}{-m_1} = 1546$ s and

$$F_1 = \tau_1\ \mathrm{e}^{b_1} = 2.28\,\mathrm{gf} \Rightarrow E_1 = \beta\, F_1 = 7.39 \times 10^5\ \mathrm{N\,m^{-2}}\ .$$

For the right graph, the final parameters are $\tau_1 = 1667$ s and $E_1 = 7.13 \times 10^5$ N m$^{-2}$ .

**D.8** 1.0pt

$\tau_1 = 1546$ s , $E_1 = 7.39 \times 10^5$ N m$^{-2}$ .




**Left:** $\frac{\mathrm{d}F}{\mathrm{d}t}$ computed numerically using equal time intervals. **Right**: using data from the graph in D.6.

## D.9

For the 4 points on the left graph in D.8, we can write

$$F(t) = F_0 + F_1\ \mathrm{e}^{-t/\tau_1} \Rightarrow F_0 = F(t) - F_1\ \mathrm{e}^{-t/\tau_1}$$

Thus, averaging $F_0$ for the 4 points of the left graph in D.8:

$$F_0 = \left(\frac{40.32 + 40.31 + 40.34 + 40.30}{4}\right) = 40.32\,\mathrm{gf}$$

Finally,

$$E_0 = \beta\, F_0 = 324293 \times 40.32\ \ \mathrm{N\,m^{-2}}\ .$$

**D.9** 0.3pt

$E_0 = 1.31 \times 10^7$ N m$^{-2}$ .

## D.10

The function $y(t)$ is given by

$$y(t) = F(t) - F_0 - F_1\, \mathrm{e}^{-t/\tau_1} \ ,$$

and was added in the Table introduced in A.3 using $F_0 = 40.32$ gf , $F_1 = 2.28$ gf and $\tau_1 = 1546$ s.

| | | |
|---|---|---|
| **D.10** | See column $y(t)$ in the Table in A.3. | 0.3pt |

## D.11

Since

$$y(t) = F(t) - F_0 - F_1\, \mathrm{e}^{-t/\tau_1} \ ,$$

then

$$y(t) = F_2\, \mathrm{e}^{-t/\tau_2} + F_3\, \mathrm{e}^{-t/\tau_3} + \cdots \quad , \quad \tau_2 > \tau_3 > \cdots$$

At long times, when the contributions from the higher components are small enough, we expect a linear behaviour for $\ln y(t)$:

$$\ln y = \ln F_2 - \frac{1}{\tau_2}\, t \ .$$

In this case, the $y(t)$ data points become meaningless above 500 s. In the region 200-500 s the graph is linear and that region can be used to extract the parameters of the second component. The equation of the straight line is $\ln y_2 = b_2 + m_2\, t$. From the graph below,

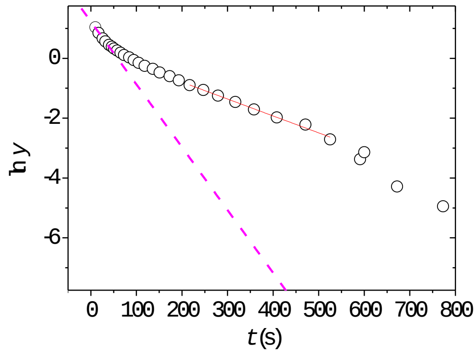
$$m_2 = -(5.65 \pm 0.19) \times 10^{-3} \ \Rightarrow \ \tau_2 = \frac{1}{-m_2} = 177\,\mathrm{s}$$

$$b_2 = 0.33 \pm 0.07 \ \Rightarrow \ F_2 = \mathrm{e}^{b_2} = 1.39 \ \Rightarrow \ E_2 = \beta\, F_2 = 4.5 \times 10^5\, \mathrm{N\,m^{-2}} \ .$$

.

**D.11** 1.0pt

$$E_2 \ = 4.5 \times 10^5\, \mathrm{N\,m^{-2}} \ , \qquad \tau_2 \ = 177\,\mathrm{s} \ .$$




The best straight line in the range 200-500 s yield the parameters $\tau_2$ and $E_2$ (Question D.11). The slope of the best straight line in the range $[10, 30]$ s give an estimate of $\tau_3$ (Questions D.12 and D.13).

## D.12

Below around 30 s there is clear deviation from a linear behaviour indicating the presence of a third component. In our case, the first data point was acquired at $t = 10$ s.

**D.12** (0.3 pt)

$$t_i = 10\,\text{s} \quad , \quad t_f = 30\,\text{s}$$

## D.13

Drawing a line in the graph using the first data points (in the range defined in D.12), as shown in the graph in D.11, $\tau_3$ can be estimated as:

$$m_3 = -0.02 \;\Rightarrow\; \tau_3 \approx m_3^{-1} \;,$$

**D.13** 0.3pt

$$\tau_3 \approx 50\,\text{s} \;.$$

### Part E. Measuring $E$ in constant stress conditions (0.6 points)

## E.1

From Question C.1 we have

$$\ell'_0 = (32.60 \pm 0.2)\,\text{cm} \;.$$

The final length $\ell'$ should be measured. In our case,

$$\ell' = 42.2 + 2 \times 0.5 = 43.2\,\text{cm} \;\Rightarrow\; \ell' = (43.2 \pm 0.2)\,\text{cm} \;.$$

Therefore, the strain is

$$\epsilon = \frac{\ell' - \ell'_0}{\ell'_0} = 0.325 \;.$$

Given that

$$E = \frac{\sigma}{\epsilon} = \frac{\frac{Mg}{\pi R^2}}{\epsilon} = \frac{80.2 \times 10^{-3} \times 9.8}{\pi \times (0.24 \times 10^{-3})^2 \times 0.325} = 1.337 \times 10^7\,\text{N}\,\text{m}^{-2} \;.$$

Note that the radius $R$ of the stretched thread was not measured. We used the value measured in task B.4: $R \approx 0.24 \times 10^{-3}$ m.

The relative difference is

$$\frac{E - E_0}{E_0} = 0.021 \; .$$

**E.1** 0.6pt

$$E \; = 1.337 \times 10^7 \, \mathrm{N\,m^{-2}} \quad , \quad \frac{E - E_0}{E_0} \; = 2.1\% \; .$$

IPhO 2018
Lisbon, Portugal

Secretariado IPhO 2018
Sociedade Portuguesa de Física
Avenida da República Nº45 3ºEsq
1050-187 Lisboa, Portugal

+351 21 799 36 65
info@ipho2018.pt

# Detailed Marking Scheme Experimental Problem 2

## Viscoelasticity of a polymer thread

(J. M. Gil, J. Pinto da Cunha, R. C. Vilão, H. V. Alberto)

July 22, 2018

v1.1

# Viscoelasticity of a polymer thread (10 points)

## Part A: Stress-relaxation measurements (1.9 points)

### A.1

| | |
|---|---|
| $\ell_0 \in [40, 50]$ cm | 0.1 |
| $\sigma_{\ell_0} \in [0.1, 0.5]$ cm | 0.2 |
| Total | 0.3 |

[Incoherent use of significant digits, $-0.1$pt]

[Wrong or missing units, $-0.1$pt]

### A.2

| | |
|---|---|
| $P_0 \in [75, 85]$ gf | 0.1 |
| $\sigma_{P_0} \in [0.01, 0.1]$ gf | 0.2 |
| Total | 0.3 |

[Incoherent use of significant digits, $-0.1$pt]

[Wrong or missing units, $-0.1$pt]

### A.3

| | |
|---|---|
| Table with times and weight | 1.0 |
| Total | 1.0 |

[Wrong or missing units in the first two columns of the table, $-0.2$pt]

[Non-decimal time scale in column $t$ (e.g. min:sec), necessary for the graphic in question D.2, $-0.2$pt]

[less than 20 points, $-0.3$pt]

[number of points $\in [20, 30]$, $-0.2$pt]

### A.4

| | |
|---|---|
| $\ell \in [45, 55]$ cm | 0.1 |
| $\sigma_{\ell} \in [0.1, 0.5]$ cm | 0.2 |
| Total | 0.3 |

[Incoherent use of significant digits, $-0.1$pt]

[Wrong or missing units, $-0.1$pt]

## Part B: Measurement of the streched thread diameter (1.5 points)

### B.1

| | |
|---|---|
| Reasonable sketch | 0.6 |
| Total | 0.6 |

[no optimization of optical path (using mirrors), $-0.6$pt]

### B.2

| | |
|---|---|
| $D \in [1, 4]$ m | 0.2 |
| $\sigma_D \in [0.1, 3]$ cm | 0.1 |
| Total | 0.3 |

[Incoherent use of significant digits, $-0.1$pt]

[Wrong or missing units, $-0.1$pt]

### B.3

| | |
|---|---|
| $\bar{x}$ | 0.1 |
| $\sigma_{\bar{x}} \in [0.001\bar{x}, 0.1\bar{x}]$ | 0.2 |
| Total | 0.3 |

[Incoherent use of significant digits, $-0.1$pt]

[Wrong or missing units, $-0.1$pt]

### B.4

| | |
|---|---|
| $d \in [0.40, 0.55]$ mm, correctly calculated from B.3 and B.2 | 0.2 |
| $\sigma_d \in [0.001, 0.05]$ mm, correctly calculated from B.3 and B.2 | 0.1 |
| Total | 0.3 |

[Incoherent use of significant digits, $-0.1$pt]

[Wrong or missing units, $-0.1$pt]

## Part C: Changing to a new thread (0.3 points)

### C.1

| | |
|---|---|
| $\ell'_0 \in [30, 35]$ cm | 0.1 |
| $\sigma_{\ell'_0} \in [0.1, 0.5]$ cm | 0.2 |
| Total | 0.3 |

[Incoherent use of significant digits, $-0.1$pt]

[Wrong or missing units, $-0.1$pt]

## Part D: Data analysis (5.7 points)

### D.1

| | |
|---|---|
| Fill $F$ in Table 1, using of the correct algorithm $F = P_0 - P(t)$ for all the calculations | 0.3 |
| Total | 0.3 |

[Errors in the calculation for some points (less than 50% of the points), $-0.1$pt]

[Errors in the calculation for some points (more than 50% of the points), $-0.3$pt]

### D.2

| | |
|---|---|
| Correct and complete representation of axis quantities, units and labels | 0.1 |
| Complete representation of all data points | 0.2 |
| Optimization of the axis span<br>in order to maximize the use of the provided space (more than half of the area) | 0.1 |
| Total | 0.4 |

**Correct and complete representation of axis quantities, units and labels**

[Missing labels in the axis, $-0.1$pt]

[Label values unequally spaced, $-0.05$ pt]

[Missing identification of the quantities in the axis, $-0.05$ pt]

[Missing or wrong units in the axis, $-0.05$ pt]

**Complete representation of all data points**

[Errors in the representation for some points (less than 50% of the points), $-0.1$ pt]

[Errors in the calculation for a significant number of points (more than 50% of the points), $-0.2$ pt]

## D.3

| | |
|---|---|
| $\epsilon$, correctly calculated from A.1 and A.4 | 0.2 |
| $\sigma_\epsilon$, correctly calculated from A.1 and B.4 | 0.1 |
| Total | 0.3 |

[Incoherent use of significant digits, $-0.1$pt]

[Indication of units for $\epsilon$, $-0.1$pt]

## D.4

| | |
|---|---|
| $\beta$, correctly calculated from D.3 and B.4 | 0.3 |
| Total | 0.3 |

[Wrong or missing units, $-0.1$pt]

## D.5

| | |
|---|---|
| Representation of a positive constant function $F(t)$ | 0.4 |
| Total | 0.4 |

## D.6

| | |
|---|---|
| Fill $\frac{\mathrm{d}F}{\mathrm{d}t}$ in Table 1 | 0.5 |
| Total | 0.5 |

[Wrong determination of d$F$/d$t$ values from either method, $-0.5$ pt]

[Use of points at t$< 1000$ s, $-0.1$ pt]

[Exclusively use of points at t$< 1000$ s, $-0.5$ pt]

## D.7

| | |
|---|---|
| Expression for expected dF(t)/dt | 0.3 |
| Total | 0.3 |

## D.8

| | |
|---|---|
| Correct and complete graphical representation of axis quantities, units and labels | 0.1 |
| Complete representation of all data points and linear fit | 0.2 |
| Optimization of the axis span<br>in order to maximize the use of the provided space (more than half of the area) | 0.1 |
| Reasonable value of $\tau_1$ | 0.3 |
| Reasonable value of $E_1$ | 0.3 |
| Total | 1.0 |

[No linearisation of d$F$/d$t$ function, $-0.4$ pt]

[Absence of a fitted straight line to extract the parameters, $-0.8$ pt]

[Bad fit of the straight line to the plotted points, $-0.3$ pt]

**Correct and complete graphical representation of axis quantities, units and labels**

[Missing labels in the axis, $-0.1$ pt]

[Label values unequally spaced, $-0.05$ pt]

[Missing identification of the quantities in the axis, $-0.05$ pt]

[Missing or wrong units in the axis, $-0.05$ pt]

**Writing reasonable value of $\tau_1$**

[Unreasonable value of $\tau_1$, expected to be in the order of $10^3$ s, $-0.2$ pt]

[Wrong or missing units in $\tau_1$, $-0.1$ pt]

**Writing reasonable value of $E_1$**

[Unreasonable value of $E_1$, expected to be in the order of $10^5$ N m$^{-2}$, $-0.2$ pt]

[Wrong or missing units in $E_1$, $-0.1$ pt]

## D.9

| | |
|---|---|
| Reasonable value of $E_0$, expected to be in the order of $1.3 \times 10^7$ N m$^{-2}$ | 0.3 |
| Total | 0.3 |

[Wrong or missing units, $-0.1$ pt]

## D.10

| | |
|---|---|
| Fill $y(t)$ in Table 1 with correct values | 0.3 |
| Total | 0.3 |

[Errors in the calculation for some points (less than 50% of the points), $-0.2$ pt]

[Errors in the calculation for a significant number of points (more than 50% of the points), $-0.3$ pt]

[Calculations for points at $t > 1000$ s, $-0.1$ pt]

## D.11

| | |
|---|---|
| Correct and complete graphical representation of axis quantities, units and labels | 0.1 |
| Complete representation of all data points and linear fit | 0.2 |
| Optimization of the axis span<br>in order to maximize the use of the provided space (more than half of the area) | 0.1 |
| Reasonable value of $\tau_2$ | 0.3 |
| Reasonable value of $E_2$ | 0.3 |
| Total | 1.0 |

[No linearisation of dF/dt function, $-0.4$ pt; Absence of a fitted straight line to extract the parameters, $-0.8$ pt; Bad fit of the straight line to the plotted points, $-0.3$ pt]

**Correct and complete graphical representation of axis quantities, units and labels**

[Missing labels in the axis, $-0.1$ pt]

[Label values unequally spaced, $-0.05$ pt]

[Missing identification of the quantities in the axis, $-0.05$ pt]

[Missing or wrong units in the axis, $-0.05$ pt]

**Writing reasonable value of $\tau_2$**

[Unreasonable value of $\tau_2$, expected to be in the order of $10^2$ s, $-0.2$ pt]

[Wrong or missing units in $\tau_2$, $-0.1$ pt]

**Writing reasonable value of $E_2$**

[Unreasonable value of $E_2$, expected to be in the order of $10^5$ N m-2, $-0.2$ pt]

[Wrong or missing units in $E_2$, $-0.1$ pt]

## D.12

| | |
|---|---|
| reasonable $t_{\text{i}}$ | 0.1 |
| Reasonable $t_{\text{f}}$ | 0.2 |
| Total | 0.3 |

[$t_{\text{f}} >$initial time in the fit of the second component, $-0.2$ pt]

[Wrong or missing units, $-0.1$ pt]

## D.13

| | |
|---|---|
| Draw a line fit within $[t_{\text{i}}, t_{\text{f}}]$ | 0.2 |
| $\tau_3$ with an order of magnitude of $10^0 - 10^1$ s ($< 100$ s) | 0.1 |
| Total | 0.3 |

[Wrong or missing units, $-0.1$ pt]

## Part E: Measuring $E$ in constant stress conditions (0.6 points)

### E.1

| | |
|---|---|
| $E$ with the same order of magnitude of that in question D.9 | 0.4 |
| Relative difference | 0.2 |
| Total | 0.6 |

[Wrong or missing units, $-0.2$ pt]