Problem 1 : Electrical conductivity in two dimensions - Answer Sheet *(10 points)*

Part A. Four-point-probe (4PP) measurements (1.2 points)

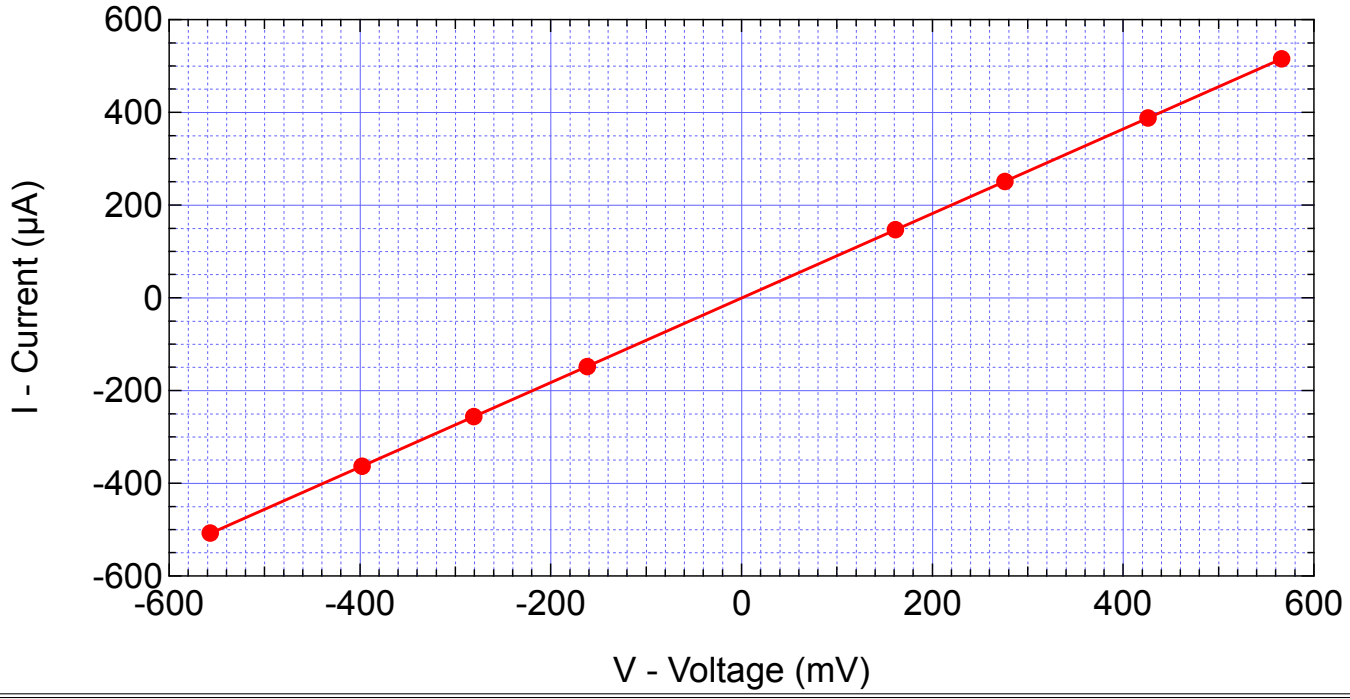
A1 (0.6 pts)

$s = 2$ cm

I (μA)	V (mV)	I (μA)	V (mV)
251	276	-148	-162
516	566	-256	-281
388	426	-363	-398
147	161	-507	-557

Plot your data in the graph B1

Graph B1: I vs V



A2 (0.2 pts)

$R = 1.08$ kΩ

A3 (0.4 pts)

$\Delta R = \pm 1\ \Omega$

Part B. Sheet resistivity (0.3 points)

B1 (0.3 pts)

$\rho_{\square} \equiv \rho_{\infty} = 4.89\ \mathrm{k\Omega}$

Part C. Measurements for different sample dimensions (3.2 points)

C1 (3 pts) and **C2 (0.2 pts)**

$s = 20$ mm

$\rho_\infty = 4.89$ kΩ

w/s	I (μA)	V (mV)	R(w/s) (kΩ)	$R_{average}$ (kΩ)		$\hat{R}$
0.3	92	1477	16.1	15.9		14.7
0.3	74	1184	16			
0.3	57	914	16			
0.3	41	651	15.9			
0.3	23	358	15.6			
0.5	154	1306	8.5	8.5		7.8
0.5	127	1079	8.5			
0.5	97	824	8.5			
0.5	67	567	8.5			
0.5	38	321	8.4			
1	233	1071	4.6	4.6		4.3
1	174	799	4.6			
1	135	621	4.6			
1	101	465	4.6			
1	59	271	4.6			
2.5	389	749	1.9	1.9		1.8
2.5	319	635	2			
2.5	237	457	1.9			
2.5	151	291	1.9			
2.5	74	143	1.9			
5	467	648	1.4	1.4		1.3
5	419	577	1.4			
5	363	499	1.4			
5	289	398	1.4			
5	185	254	1.4			

Part D. Geometrical correction factor (1.9 points)

D1 (1.0 pts)

Plot your data on the appropriate graph paper: linear (Graph E1a), semi-logarithmic (D1b) **or** double-logarithmic (D1c) on the following pages.

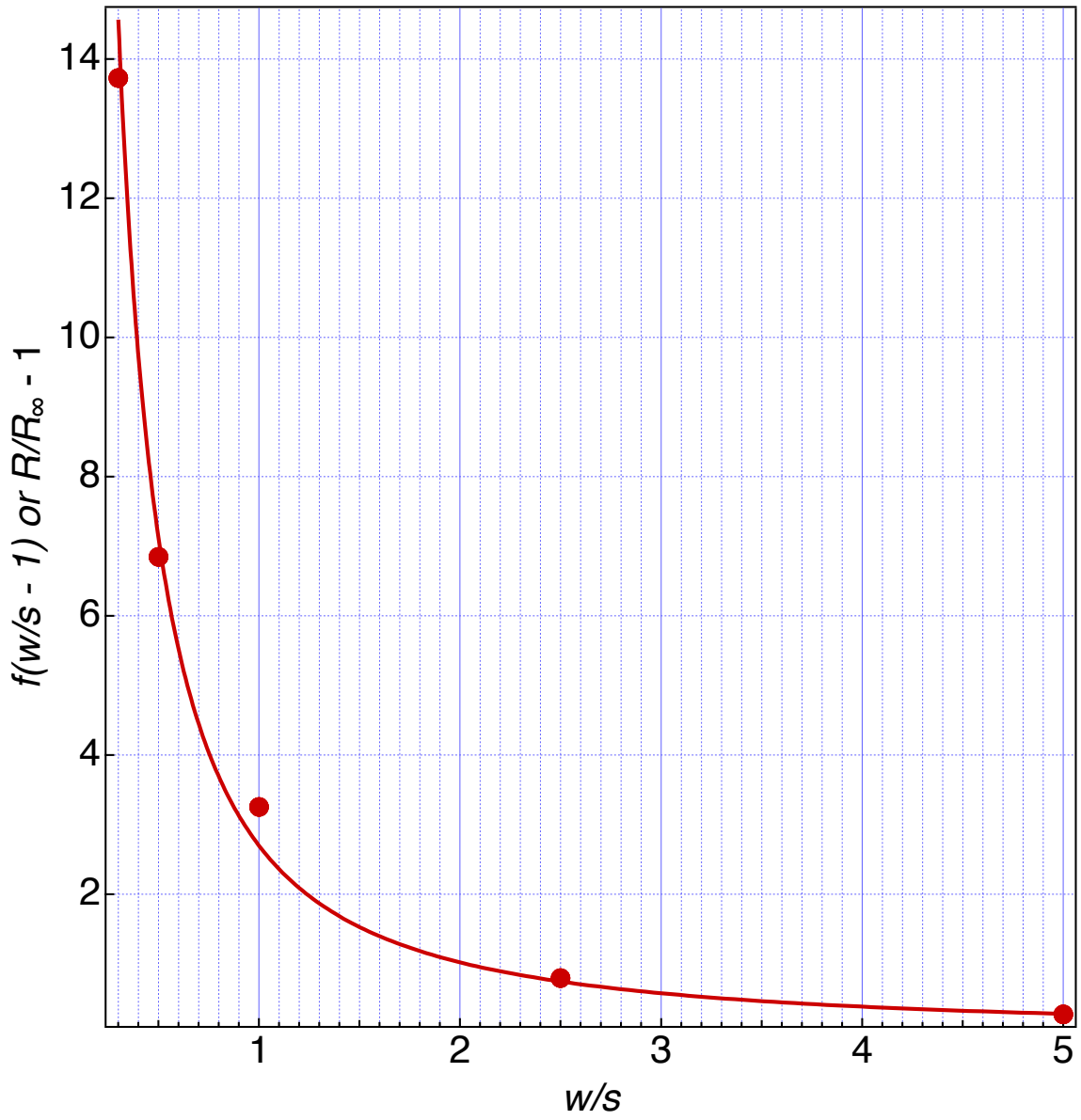
D2 (0.9 pts)

$a = 2.7$

$b = -1.4$

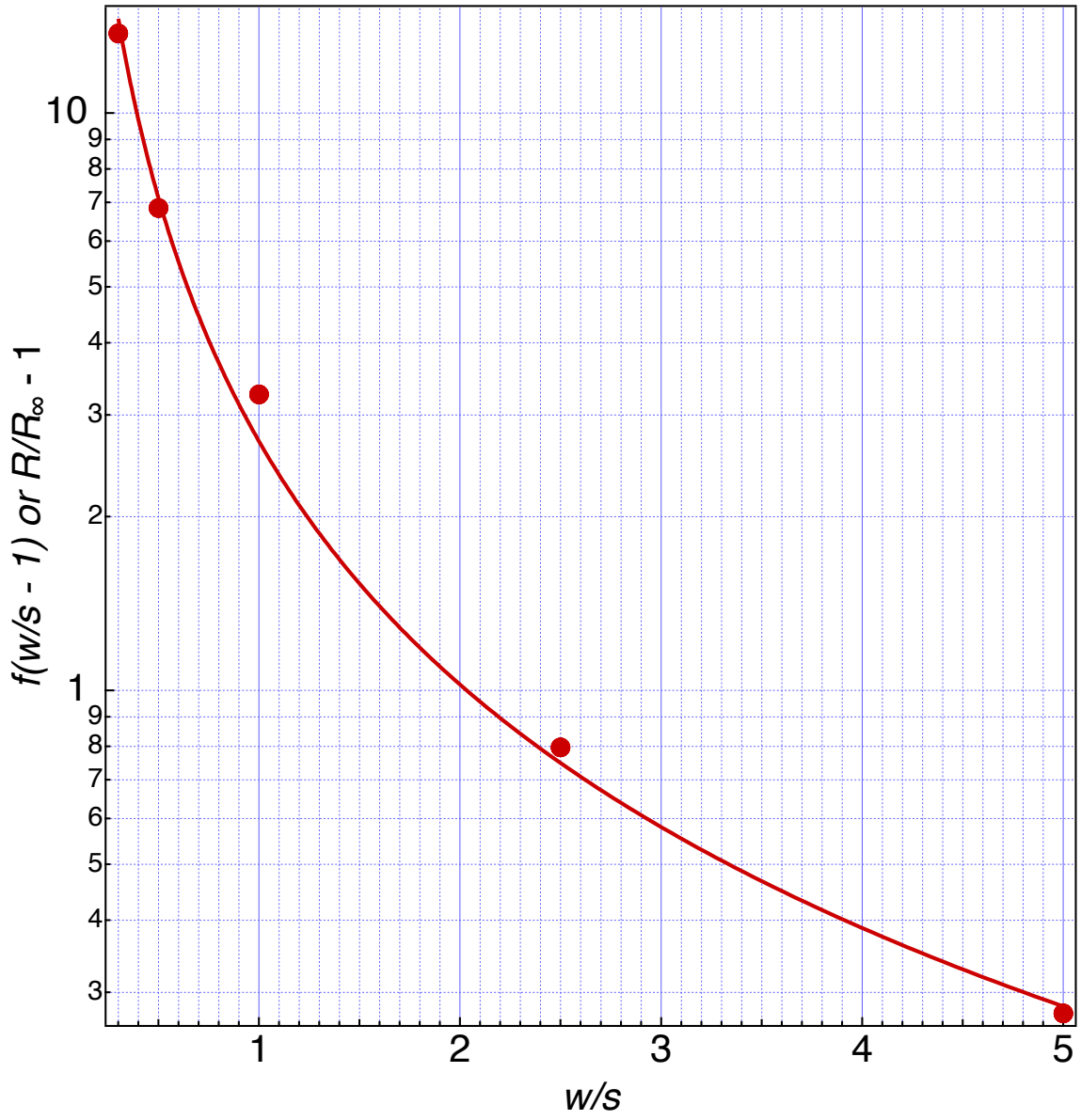
Graph D1a: linear scale: I vs V

Wrong. The usage of linear scale does not allow for deduction of the parameters.



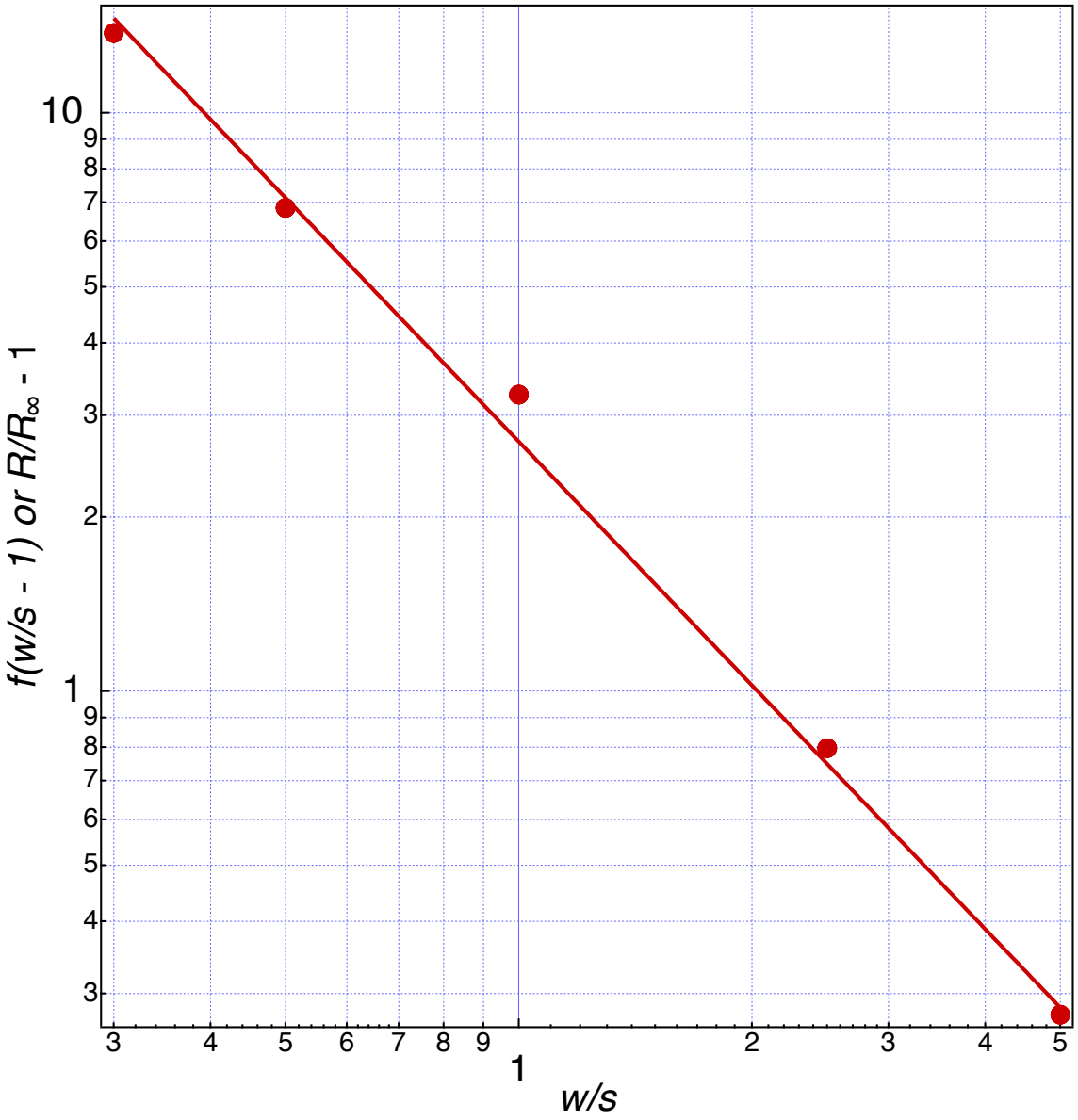
Graph D1b: semi-log scale: I vs V

Wrong. The usage of semi-log scale does not allow for deduction of the parameters.



Graph D1c: double-log scale: I vs V

Correct. The parameters can be deduced by fitting a line.



Part E. van der Pauw-method (3.4 points)

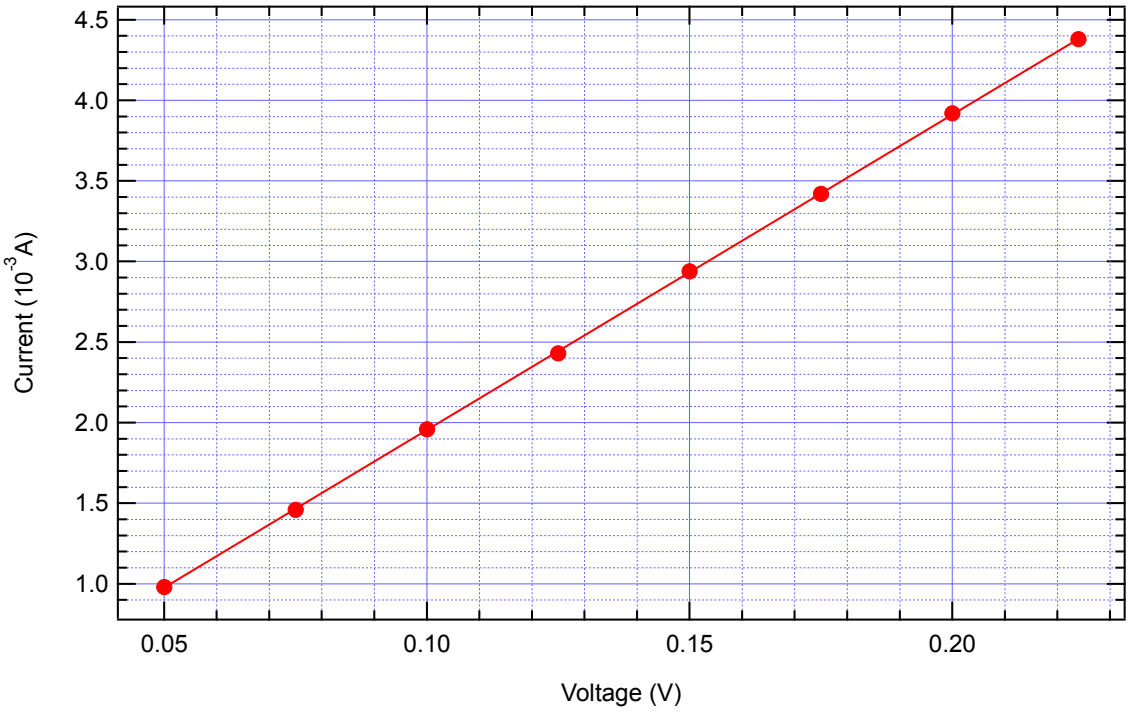
Note the number of your wafer here: 99 (between 1 - 450)

E1 (0.4 pts)

I (mA)	V (mV)	I (mA)	V (mV)
0.98	50	2.94	150
1.46	75	3.42	175
1.96	100	3.92	200
2.43	125	4.38	224

E2 (0.4 pts)

Graph F2: I vs V



$R_{4PP} = 51.1\Omega$

E3 (0.2 pts)

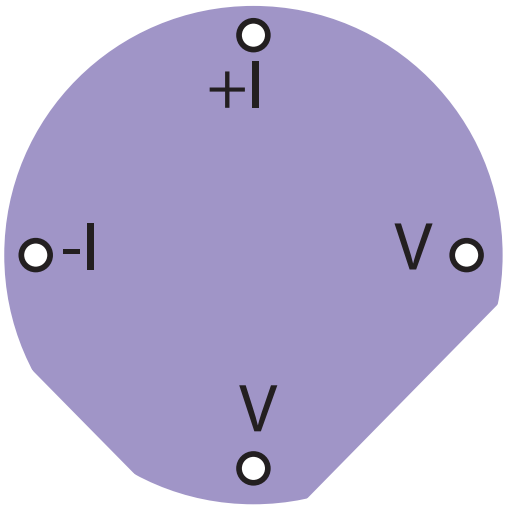
$w = 10$ cm $\rightarrow$ $w/s = 5$ $f(w/s) = 1.284$

E4 (0.1 pts)

$\rho_{\square} = 180\,\Omega$

E5 (0.6 pts)

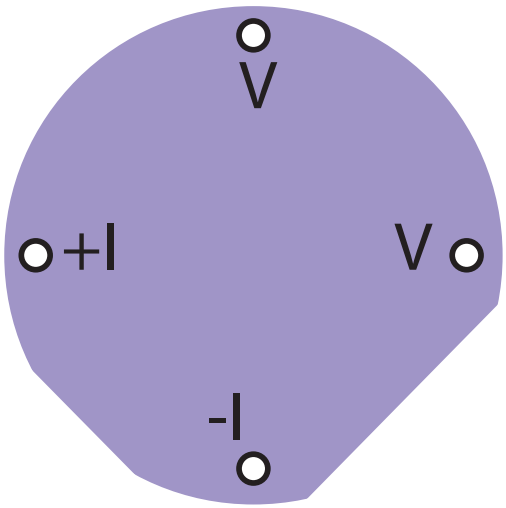
Sketch (orientation of the current):



V mV	I mA
140	3.71
120	3.18
100	2.66
80	2.12
60	1.58
40	1.06
20	0.53
-20	-0.54
-40	-1.06
-60	-1.61
-80	-2.13
-100	-2.68
-120	-3.2
-136	-3.62

E6 (0.6 pts)

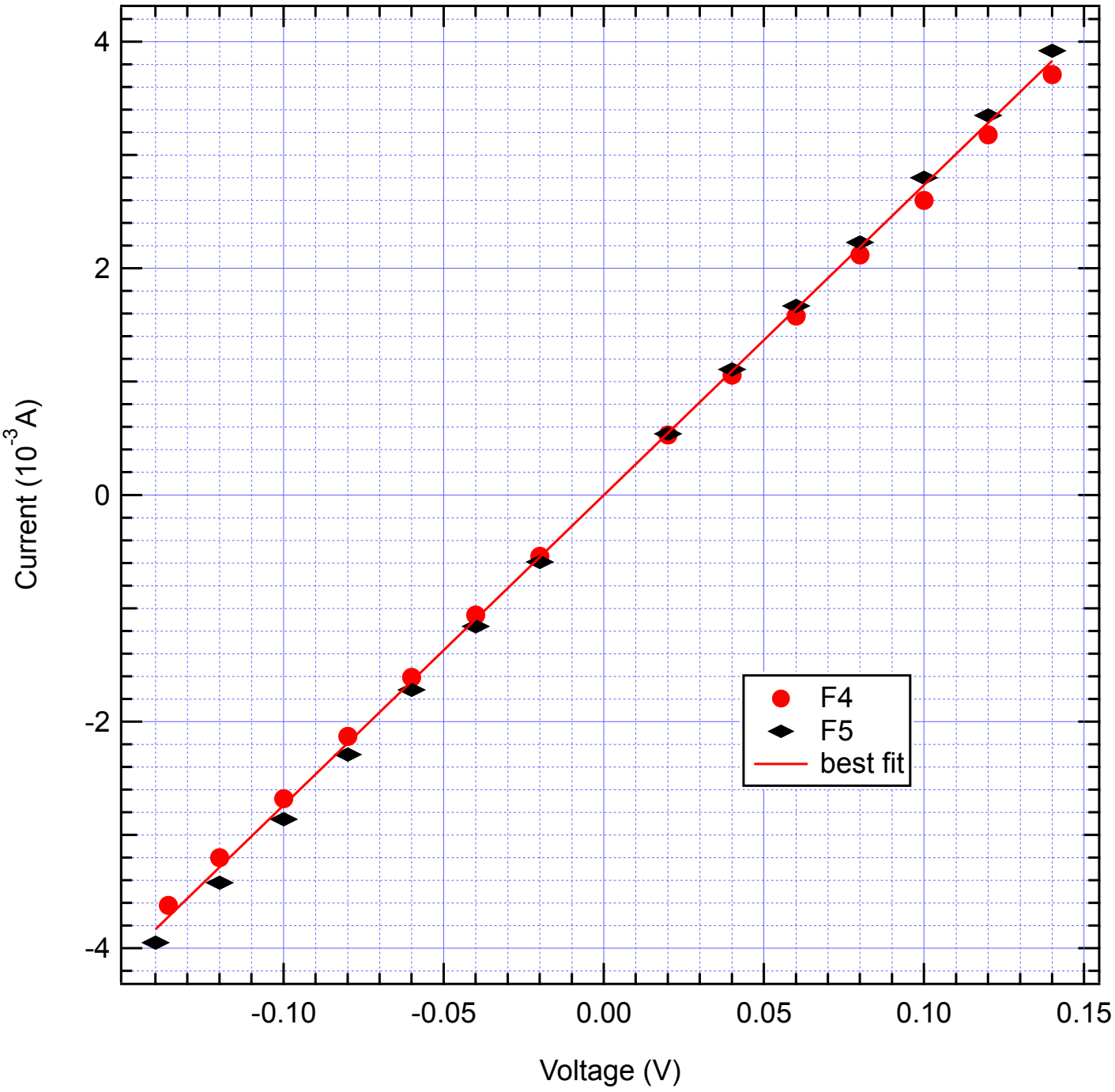
Sketch (orientation of the current):



I	V
140	3.92
120	3.35
100	2.8
80	2.23
60	1.67
40	1.11
20	0.54
-20	-0.59
-40	-1.16
-60	-1.72
-80	-2.29
-100	-2.86
-120	-3.42
-140	-3.95

E7 (0.5 pts)

Graph F6: I vs V



$\langle R \rangle = 36.5\ \Omega$

E8 (0.4 pts) Calculation:

$2 \cdot e^{-\pi \cdot \langle R \rangle / \rho_\square} = 1$ $\quad$ $e^{-\pi \cdot \langle R \rangle / \rho_\square} = 1/2$

$-\dfrac{\pi \cdot \langle R \rangle}{\rho_\square} = \ln(1/2)$ $\quad$ $\dfrac{\pi \cdot \langle R \rangle}{\rho_\square} = \ln(2)$

$\rho_\square = \dfrac{\pi \cdot \langle R \rangle}{\ln(2)}$

$\rho_\square = 165\,\Omega$

E9 (0.1 pts)

$\frac{\Delta \rho_\square}{\rho_\square} =$ 0.091 $=$ 9.1 %

E10 (0.1 pts)

Resistivity of the Cr thin film $\rho = 1.32 \cdot 10^{-6}\,\Omega \cdot \mathrm{m}$

Experiment: "Electrical conductivity in two dimensions"

Marking Scheme

Part			Maximum points		Total points
A: Four-point-probe (4PP) measurements					1.2
A1	Value	$1.9 < s < 2.1$	0.1	0.6	
	Table and Graph	I and V are measured at 4 or more points	0.3		
		Points are properly marked using the majority part of the graph	0.2		
		If I > V	-0.1		
		Missing or incorrect units (either or both)	-0.1		
		Missing axis labels (either or both)	-0.1		
A2	Calculation	1000 Ohm < R < 1200 Ohm with units	0.2	0.2	
A3	Calculation	Either: extremal lines with slopes, error= difference of slopes, or numerical regression analysis. If the dispersion of measurements from the mean line not visible, error propagation from instrument error is allowed or a conclusion that error is negligible.	0.4	0.4	
		Missing or incorrect units (either or both)	-0.1		
B: Sheet resistivity					0.3
B1	Calculation	$\rho_{\square}$ calculation is consistent with A2	0.3	0.3	
		Missing or incorrect units (either or both)	-0.1		
C: Measurements for different sample dimensions					
C1	Measurements	4 values w/s, ≥ 4 data points per w/s	3	3	3.2
		3 values 4 w/s	2		
		2 or less values 4 w/s	0		
		4 or 3 values w/s, ≥ 3 data points per w/s	-0.5		
		4 or 3 values w/s, ≥ 2 data points per w/s	-1		
		4 or 3 values w/s, ≥ 1 data points per w/s	-1.5		

Part			Maximum points		Total points
		Missing or incorrect units (either or both)	-0.1		
C2	Calculation	*f(w/s) for* ≥ 3 values	0.2	0.2	
		f(w/s) for 1 or 2 values	0.1		
D: Geometrical correction factor					1.9
D1	Graph	Choice of appropriate graph and axis values, so that the marks should lie on a line.	0.8	1	
		Points are properly marked using the major part of the graph; irrespective of graph used	0.2		
		Missing or incorrect labels (either or both)	-0.1		
		Missing or incorrect units (either or both)	-0.1		
D2		Reasonable fit over all marks	0.3	0.9	
		2.2 < a < 3.6	0.3		
		1.8 < a < 2.2, 3.6 < a < 4	0.2		
		-2 < b < -1	0.3		
		Missing or incorrect labels (either or both)	-0.1		
		Missing or incorrect units (either or both)	-0.1		
E: The silicon wafer and van der Pauw-method					3.4
E1	Table	0.1 per I and V measurement; max 0.4 points	0.4	0.4	
		Missing or incorrect units (either or both)	-0.1		
E2	Graph and Calculation	Points are properly marked using the majority part of the graph	0.1	0.4	
		Reasonable fit over all marks	0.1		
		R_{4PP} according to the wafer table ± 15 %, if wafer number is not known use R_{4PP} = 55 Ω	0.2		

Part			Maximum points		Total points
		R_{4PP} according to the wafer table 15.1 30 %, if wafer number is not known use R_{4PP} = 55 Ω	0.1		
		Missing or incorrect labels (either or both)	-0.1		
		Missing or incorrect units (either or both)	-0.1		
E3	Calculation	Consistent calculation of f(w/s) with D2	0.2	0.2	
E4	Calculation	$\rho_{\square}$(4PP)	0.1	0.1	
		No or incorrect units	-0.1		
E5	Sketch and table	Sketch present and makes sense	0.2	0.6	
		6 different I and V values are taken	0.4		
		5 different I and V values are taken	0.3		
		4 different I and V values are taken	0.2		
		3 different I and V values are taken	0.1		
		2 or less different I and V values are taken	0		
		V points are extremely unequally spaced	-0.1		
		Missing or incorrect units (either or both)	-0.1		
E6	Sketch and table	Sketch should be perpendicular to F5, otherwise the whole part gets 0 points		0.6	
		Sketch present, makes sense and perpendicular to the sketch of F5	0.2		
		6 different I and V values are taken	0.4		
		5 different I and V values are taken	0.3		
		4 different I and V values are taken	0.2		
		3 different I and V values are taken	0.1		
		2 or less different I and V values are taken	0		
		V points are extremely unequally spaced	-0.1		
		Missing or incorrect units (either or both)	-0.1		

Part			Maximum points		Total points
E7	Graph	Points are properly marked using the majority part of the graph	0.1	0.5	
		Reasonable fit over all marks	0.1		
		$\langle R \rangle$ = vdPauw resistance of wafer table ± 10 %, if wafer number is not known use $\langle R \rangle$ = 42 Ω	0.3		
		$\langle R \rangle$ = vdPauw resistance of wafer table ± 10.1 to 20 %, if wafer number is not known use $\langle R \rangle$ = 42 Ω	0.2		
		$\langle R \rangle$ = vdPauw resistance of wafer table ± 20.1 to 30 %, if wafer number is not known use $\langle R \rangle$ = 42 Ω	0.1		
		Missing or incorrect labels (either or both)	-0.1		
		Missing or incorrect units (either or both)	-0.1		
E8	Solve Eqn.	$\rho_{\square} = \frac{\pi}{\ln 2} \langle R \rangle$ formula is present	0.3	0.4	
	Calculation	Consistent calculation $\rho_{\square}$	0.1		
		Missing or incorrect units	-0.1		
E9	Calculation	Value is written with correct units (fraction, decimal and % are accepted)	0.1	0.1	
E10	Calculation	Consistent calculation ρ	0.1	0.1	
		Missing or incorrect units	-0.1		

Total number of points	10

Problem 2 : Solution – Jumping Beads - a model for phase transitions and instabilities *(10 points)*

Part A. Critical driving amplitude (3.3 points)

A1 (1.2 pts)

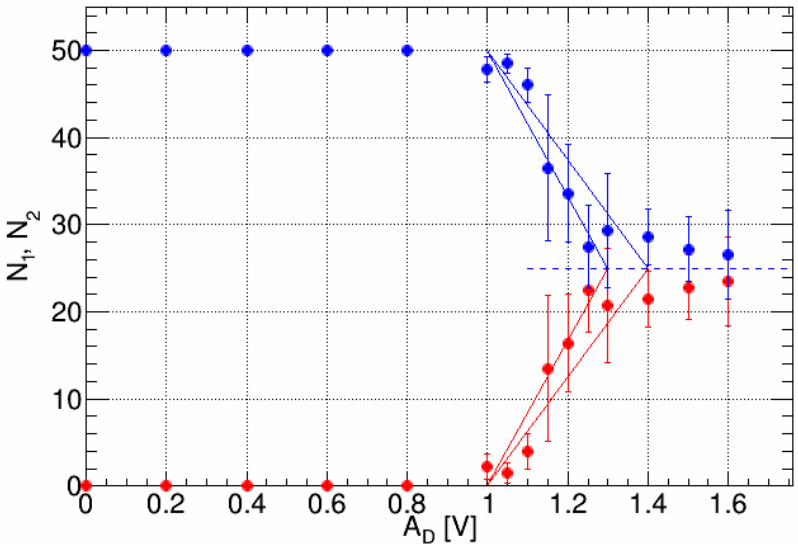
Total number of seeds: $N_0 = 50$.

Number of readings: $n = 6$.

A_D, [V]	N_1						$\bar{N}_1 = \frac{1}{n}\Sigma_{i=1}^{n} N_1^i$	$\bar{N}_2 = N_0 - \bar{N}_1$	$\sigma = \sqrt{\frac{\Sigma_{i=1}^{n}(N_i - \bar{N})^2}{n-1}}$	SE $= \frac{\sigma}{\sqrt{n}}$
1.00	1	5	2	1	2	2	**2.2**	**47.8**	**1.5**	**0.6**
1.05	1	0	2	3	1	2	**1.5**	**48.5**	**1.1**	**0.5**
1.10	4	4	1	7	3	5	**4.0**	**46.0**	**2.0**	**0.8**
1.15	26	5	18	7	18	7	**13.5**	**36.5**	**8.4**	**3.4**
1.20	13	16	27	12	17	13	**16.4**	**33.7**	**5.6**	**2.3**
1.25	26	28	22	22	14	23	**22.5**	**27.5**	**4.8**	**2.0**
1.30	27	24	8	22	22	21	**20.7**	**29.3**	**6.6**	**2.7**
1.40	22	18	17	23	23	25	**21.4**	**28.7**	**3.2**	**1.3**
1.50	19	27	27	24	19	21	**22.8**	**27.2**	**3.7**	**1.5**
1.60	27	15	23	23	23	30	**23.5**	**26.5**	**5.1**	**2.1**

Plot the data in the graph A2.

A2 (1.1 pts)



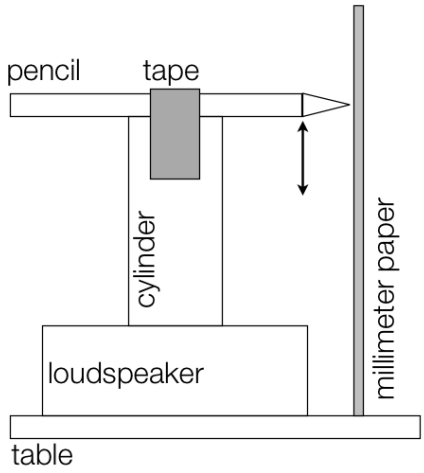
Error bars represent either standard deviation (σ) or standard error (SE).

A3 (1.0 pts)

$A_{D,\text{crit}} = (1.25 \pm 0.05)$ V

Part B. Calibration (3.2 points)

B1 (0.5 pts)

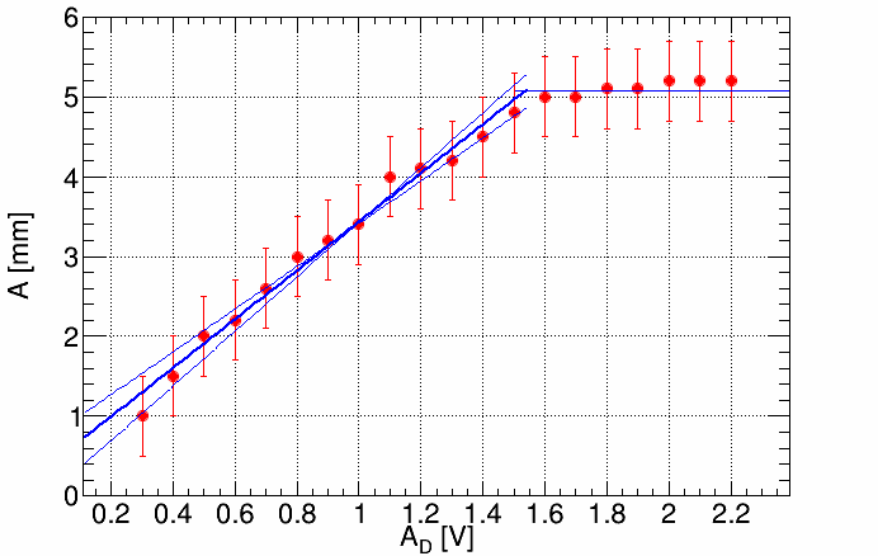


B2 (0.8 pts)

A_D [V]	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1
A [mm]	1.0	1.5	2.0	2.2	2.6	3.0	3.2	3.4	4.0	4.1	4.2	4.5	4.8	5.0	5.0	5.1	5.1	5.2	5.2

Instrumental error ± 0.5 mm.

B3 (1.0 pts)



B4 (0.8 pts)

$A = k_0 + k_1 \times A_D$,
where:
$k_0 = 0.2$ [mm], $k_1 = 3.1$ [mm/V]

B5 (0.1 pts)

$A_{\text{crit}} = (4.4 \pm 0.1)$ mm

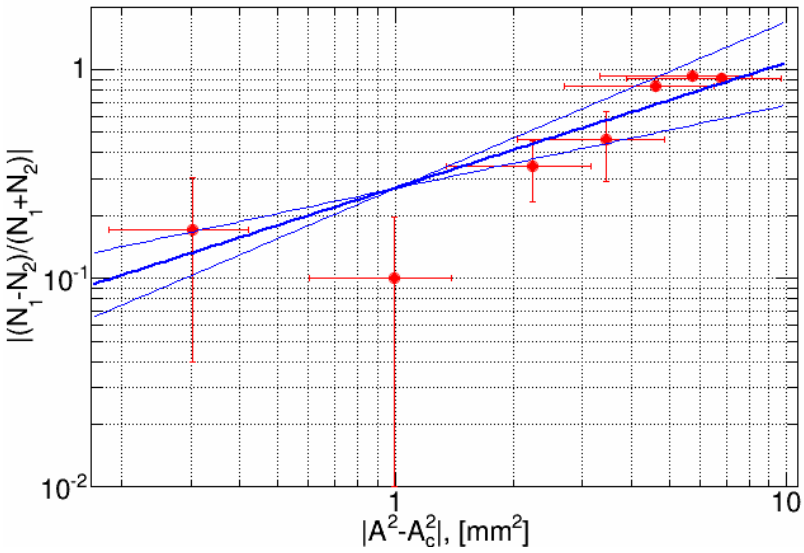
Part C. Critical exponent (3.5 points)

C1 (1.1 pts)

A_D, [V]	A, [mm]	$\lvert\frac{N_1-N_2}{N_1+N_2}\rvert$	$\lvert A^2 - A_0^2\rvert$
1.00	3.5	0.91	6.8
1.05	3.6	0.94	5.7
1.10	3.8	0.84	4.6
1.15	3.9	0.46	3.5
1.20	4.1	0.35	2.2
1.25	4.2	0.10	1.0
1.30	4.4	0.17	0.3
1.40	4.7	0.15	
1.50	5.0	0.09	
1.60	5.3	0.06	

Plot the data in the graph C2.

C2 (1.0 pts)



C3 (1.4 pts)

$y = a \cdot x^b$, where $x = |A^2 - A_0^2|$, $y = |\frac{N_1-N_2}{N_1+N_2}|$.

Critical exponent $b = 0.6 \pm 0.2$.

E2: Jumping Beads - a model for phase transitions and instabilities

Marking Scheme

Part			Maximum points		Total points
A:	Determination of the critical driving amplitude for the loudspeaker				3.3
A1	Observation table	Measurements covering the critical range with 9 or more readings at different settings of the amplitude	0.2	1.2	
		Measurements covering the critical range with 5 to 8 readings at different settings of the amplitude	0.1		
		Measurement interval at least 100mV	0.1		
		Interval smaller at the critical point (50mV)	0.2		
		Proper table header	0.1		
		Writing unit for X	0.2		
		Statistics: 5 or more readings at the same amplitude	0.3		
		Average N_2 derived from N_1	0.1		
A2	Graph	Computed error and written uncertainty for each measured data point	0.2	1.1	
		Proper choice of scale for X axis (at least half of the full width of the paper used and all data points fit in the plot)	0.1		
		Proper choice of scale for Y axis (at least half of the full height of the paper used and all data points fit in the plot)	0.1		
		Variables written along axes	0.2		
		Units mentioned (X axis)	0.1		
		Correct plotting of points	0.2		
		Plotted uncertainty	0.2		
A3	Calculations	Critical amplitude determined in a reasonable way	0.4	1	
		The range for the determination is correct (flat parts far from the critical region not included)	0.3		

Part			Maximum points		Total points
		Result is within (1.0V ≤ $A_{D,\,crit}$ ≤ 1.5V)	0.3		
		Result is within (0.7V ≤ $A_{D,\,crit}$ <1.0V)	0.1		
		Result is within (1.5V < $A_{D,\,crit}$ ≤ 1.7V)	0.1		
B: Calibration of the loudspeaker driving amplitude					3.2
B1	Sketch	Sketch present and makes sense (ignoring the text description)	0.5	0.5	
B2	Table	Proper table header	0.2	0.8	
		Units mentioned	0.2		
		Uncertainty is written and is in the reasonable range (0.3 to 1mm)	0.2		
		At least 5 data points in the proper (linear) range	0.2		
B3	Graph	Proper choice of scale for X axis (at least half of the full width of the paper used and all data points fit in the plot)	0.1	1	
		Proper choice of scale for Y axis (at least half of the full height of the paper used and all data points fit in the plot)	0.1		
		Variables written along axes	0.2		
		Units mentioned	0.2		
		Correct plotting of points	0.2		
		Plotted uncertainty as error bars	0.2		
B4	Calculation	Fit present on the plot in the correct range (plateau not included)	0.2	0.8	
		Functional form written	0.1		
		Linear function is used for the fit	0.1		
		Slope written and within range (2.5–3.5)	0.1		
		Written unit for the slope [mm/V]	0.1		
		Offset written and within range (–0.5 to +0.5)	0.1		
		Written unit for the offset [mm]	0.1		

Part			Maximum points		Total points
B5	Critical amplitude	Critical driving amplitude computed using computed calibration curves	0.1	0.1	
C: Critical exponent					3.5
C1	Table	Proper table header	0.1	1.1	
		Writing unit for X	0.2		
		Imbalance calculated correctly (ranges from 0 to 1)	0.2		
		$\lvert A^2 - A_c^2 \rvert$ computed for the driving amplitude of the speaker (in mm), not for the excitation amplitude of the signal (in V)	0.2		
		Proper conversion from A_D to A using calibration curve determined in Part B	0.2		
		Maximum value of imbalance is at least 10 times larger than its minimum value	0.2		
C2	Graph	Double-logarithmic paper used correctly (or logarithm computed and normal paper used)	0.2	1	
		Scale chosen appropriately (at least half of the full width/height of the paper used and all data points fit in the plot)	0.2		
		Variables written along axes	0.2		
		Units present (X axis)	0.2		
		Correct plotting of points	0.2		
C3	Calculation	Fit does not include values where $A > A_c$	0.3	1.4	
		Method to determine the critical exponent b is correct (understanding that it is related to the slope of the straight line in double-logarithmic plot, hence line is drawn)	0.3		
		The method to calculate the slope is correct	0.2		
		Exponent within range (0.0–1.0)	0.2		
		Error written	0.2		
		Error within range (0.1–0.5)	0.2		
Total number of points:					10