Part 1. Calibration

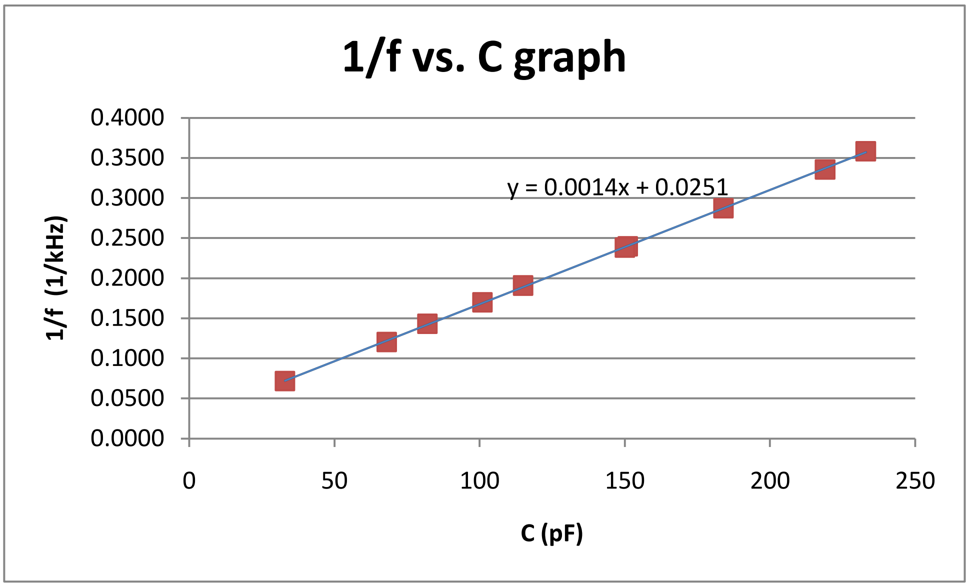
From the relationship between f and C given,

$$f = \frac{\alpha}{C + C_s} \qquad \Leftrightarrow \qquad \frac{1}{f} = \frac{1}{\alpha}C + \frac{C_s}{\alpha}$$

That is, theoretically, the graph of $\frac{1}{f}$ on the Y-axis versus C on the X-axis should be linear of which the slope and the Y-intercept is $\frac{1}{\alpha}$ and $\frac{C_s}{\alpha}$ respectively.

The table below shows the measured values of C (plotted on the X-axis,) f and, additionally, $\frac{1}{f}$, which is plotted on the Y-axis.

C (pF)	f (kHz)	1/f (ms)
33	13.94	0.0717
68	8.30	0.1205
82	6.99	0.1431
151	4.17	0.2398
233	2.79	0.3584
219	2.98	0.3356
184	3.48	0.2874
150	4.20	0.2381
115	5.24	0.1908
101	5.89	0.1698



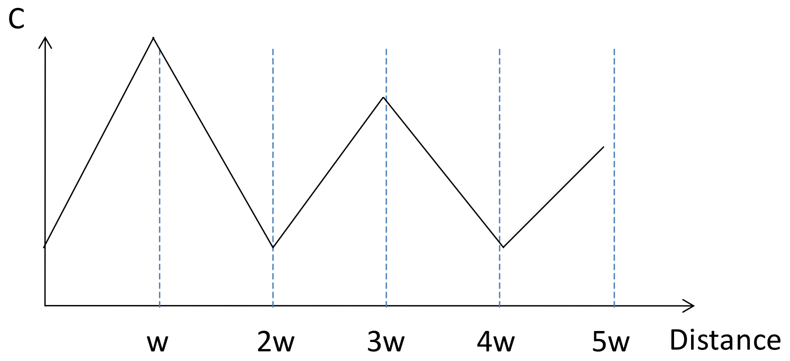
From this graph, the slope ($\frac{1}{\alpha}$) and the Y-intercept ($\frac{C_s}{\alpha}$) is equal to 0.0014 s/nF and 0.0251 ms respectively.

Hence, $$\alpha = \frac{1}{\text{slope}} = \frac{1}{0.0014\ \text{s}/\text{nF}} = 714\ \text{nF/s}$$

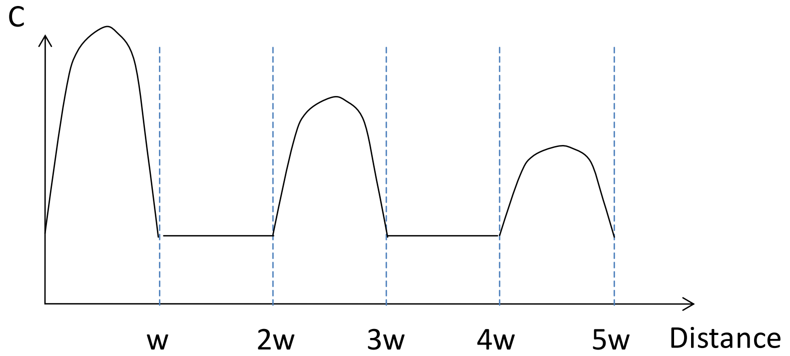
and $$C_s = \frac{\text{Y} - \text{intercept}}{\text{slope}} = \frac{0.0251\ \text{ms}}{0.0014\ \text{s}/\text{nF}} = 17.9\ \text{pF}$$ as required.

Part II. Determination of geometrical shape of parallel-plates capacitor

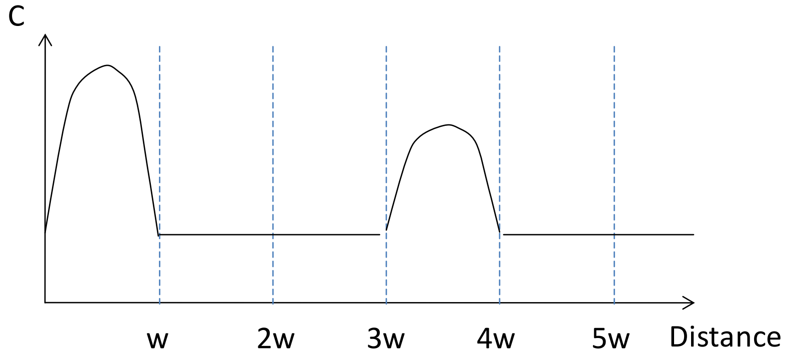
PATTERN I: The expected graph of C versus the position



PATTERN II: The expected graph of C versus the position

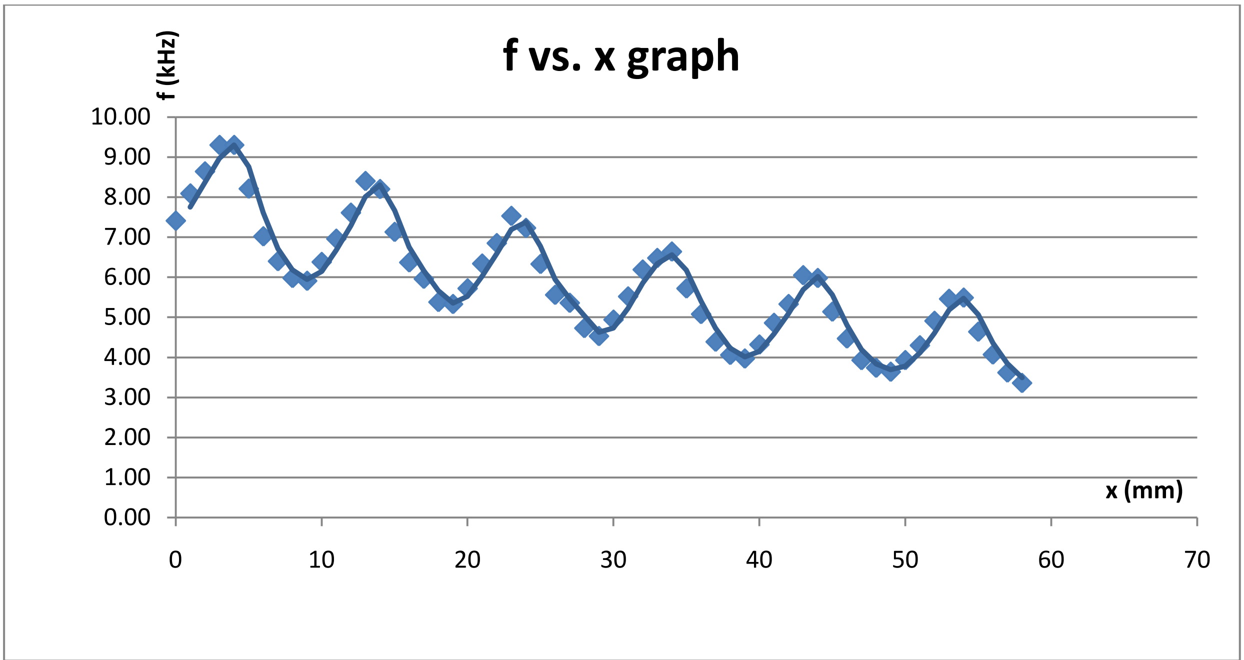


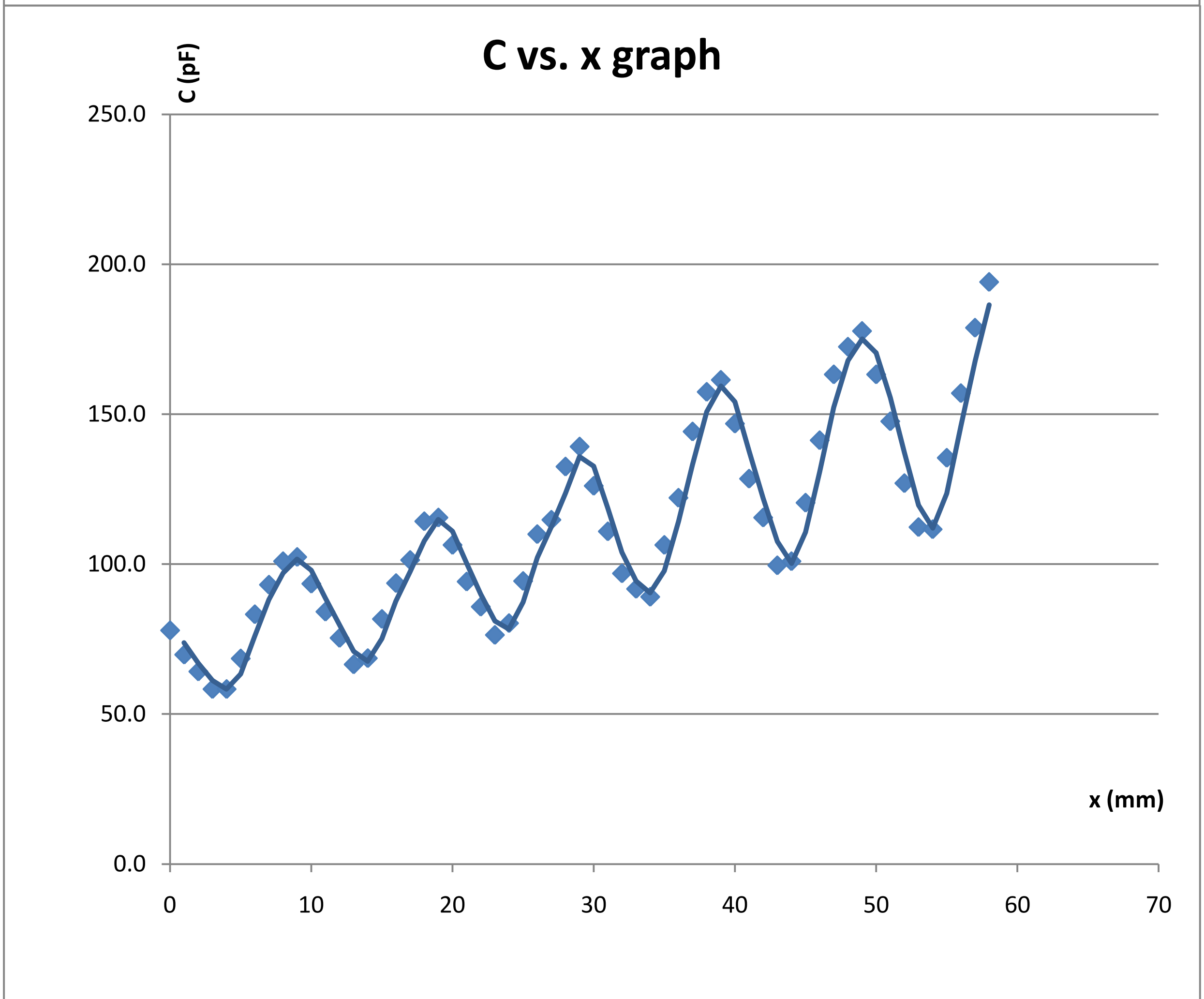
PATTERN III: The expected graph of C versus the position



By measuring f and C versus x (the distance moved between the two plates,) the data and the graphs are shown below.

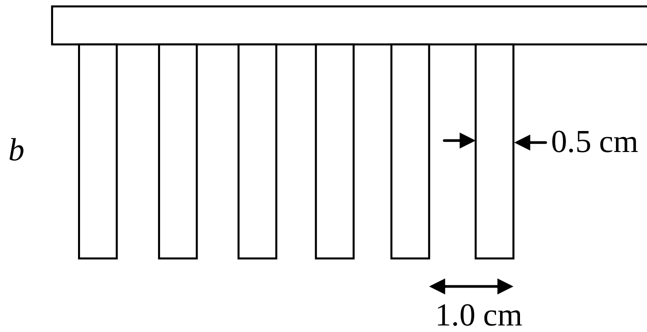
x (mm)	f (kHz)	C (pF)	x (mm)	f (kHz)	C (pF)
0	7.41	77.9	30	4.94	126.1
1	8.09	69.8	31	5.52	110.9
2	8.64	64.2	32	6.19	96.9
3	9.30	58.3	33	6.48	91.7
4	9.30	58.3	34	6.64	89.1
5	8.21	68.5	35	5.72	106.4
6	7.02	83.3	36	5.08	122.1
7	6.40	93.1	37	4.39	144.2
8	5.98	100.9	38	4.06	157.4
9	5.91	102.4	39	3.97	161.4
10	6.38	93.5	40	4.32	146.8
11	6.96	84.1	41	4.86	128.5
12	7.61	75.4	42	5.33	115.5
13	8.40	66.5	43	6.05	99.6
14	8.20	68.6	44	5.98	100.9
15	7.13	81.7	45	5.14	120.5
16	6.37	93.6	46	4.47	141.3
17	5.96	101.3	47	3.93	163.3
18	5.38	114.3	48	3.74	172.5
19	5.33	115.5	49	3.64	177.7
20	5.72	106.4	50	3.93	163.3
21	6.34	94.2	51	4.30	147.6
22	6.85	85.8	52	4.91	127.0
23	7.53	76.4	53	5.46	112.3
24	7.23	80.3	54	5.49	111.6
25	6.33	94.3	55	4.64	135.4
26	5.56	110.0	56	4.07	157.0
27	5.36	114.8	57	3.62	178.8
28	4.73	132.5	58	3.36	194.1
29	4.53	139.2			


f vs. x graph
f (kHz)
10.00
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
x (mm)
0
10
20
30
40
50
60
70


C vs. x graph
C (pF)
250.0
200.0
150.0
100.0
50.0
0.0
x (mm)
0
10
20
30
40
50
60
70

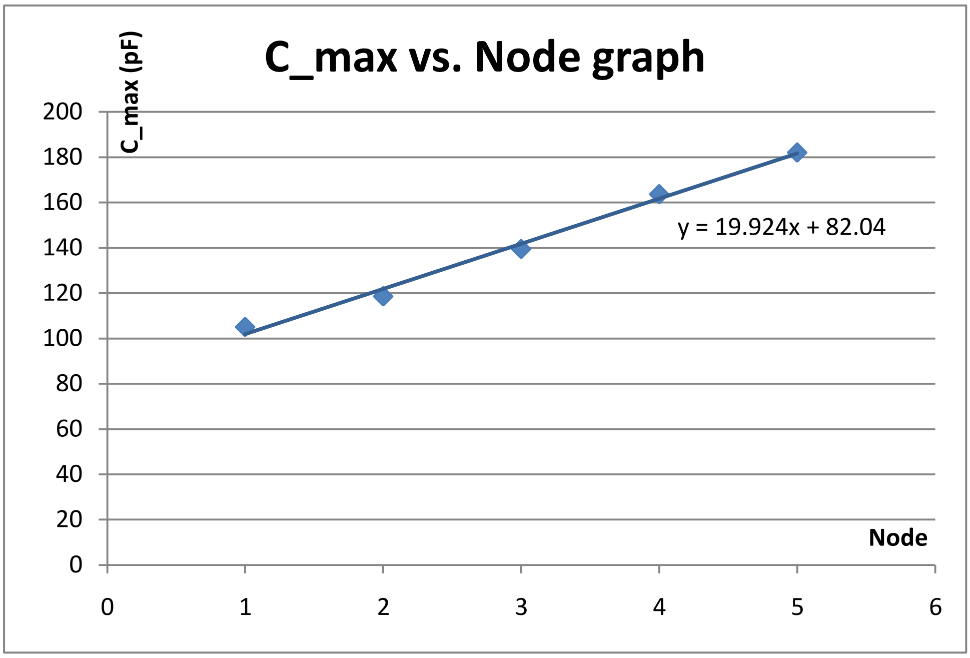
From periodicity of the graph, period = 1.0 cm

Simple possible configuration is:



The peaks of C values obtained from the C vs. x graph are provided in the table below. These maximum C are plotted (on the Y-axis) vs. nodes (on the X-axis.)

node	C_max
1	105.1
2	118.6
3	139.5
4	163.7
5	182.1



This graph is linear of which the slope is the dropped off capacitance $\Delta C = 19.9$ pF/section.

Given that the distance between the plates $d = 0.20$ mm, $K = 1.5$,

$$\Delta C \approx \frac{K\varepsilon_0 A}{d},$$

and $A = 5\times 10^{-3}\,\mathrm{m} \times b\,\mathrm{mm} \times 10^{-3}\ \mathrm{m}^2$

Then, $b\ \text{mm} \approx \dfrac{\Delta C\ d}{K\varepsilon_0 \times 10^{-3} \times 5 \times 10^{-3}} \approx 60\ \text{mm}$ if medium between plates is the dielectric of which $K = 1.5$.

Part III. Resolution of digital micrometer

From the given relationship between f and C, $f = \dfrac{\alpha}{C + C_S}$,

$$\Delta f \simeq \left|\frac{df}{dC}\right| \Delta C = \left|\frac{-\alpha}{(C + C_S)^2}\right| \Delta C$$

$$= \frac{f^2}{\alpha} \Delta C$$

$$\Leftrightarrow \qquad \Delta C = \frac{\alpha}{f^2} \Delta f$$

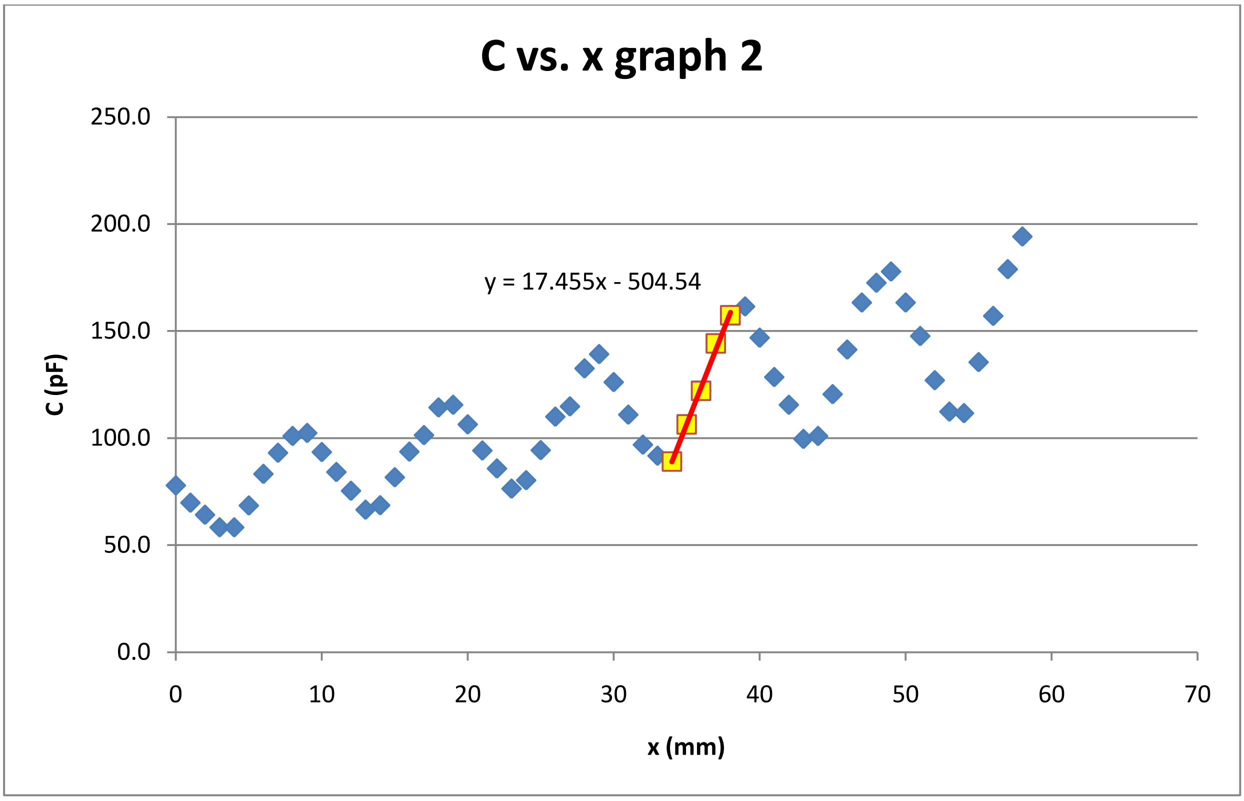
And since C linearly depends on x, $C = mx + \beta \quad \Rightarrow \quad \Delta C = m\Delta x$.

Hence,

$$\Delta x = \frac{\alpha}{mf^2} \Delta f,$$

where Δf is the smallest change of the frequency f which can be detected by the multimeter, x_0 is the operated distance at f = 5 kHz, and m is the gradient of the C vs. x graph at $x = x_0$.

From the f vs. x graph, at f = 5 kHz, The gradient is then measured on the C vs. x graph around this range.



From this graph, $m = 17.5\ \mathrm{pF/mm} = 1.75\times10^{-8}\,\mathrm{F/m}$.
Using this value of m, $f = 5\ \mathrm{kHz}$, $\alpha = 714\ \mathrm{nF/s}$, and $\Delta f = 0.01\ \mathrm{kHz}$,

$$\Delta x = \frac{714\times10^{-9}}{(1.75\times10^{-8})(5\times10^{3})^{2}}\times(0.01\times10^{3}) = 0.016\,\mathrm{mm}$$

NB. The C vs. x graph is used since C (but not f) is linearly related to x.

Alternative method for finding the resolution
(not strictly correct)

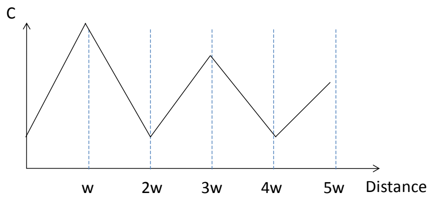
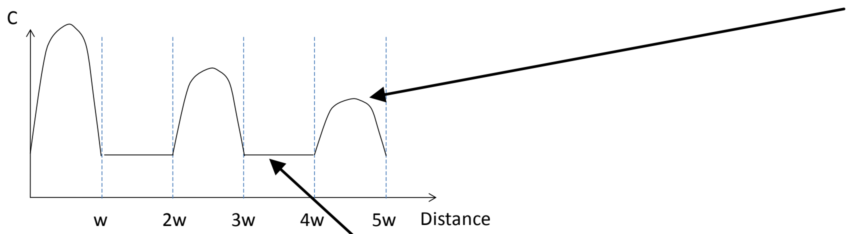
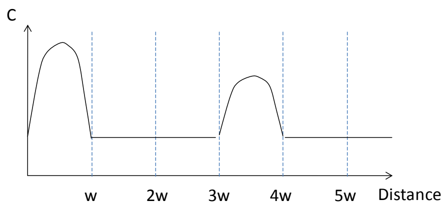
Using the f vs. x graph and the data in the table around $f = 5\ \mathrm{kHz}$, it is found that when f is changed by 1 kHz ($\Delta f = 1\ \mathrm{kHz}$,) x is roughly changed by 1.5 mm ($\Delta x \simeq 1.5\ \mathrm{mm}$.) Hence, when f is changed by $\Delta f = 0.01\ \mathrm{kHz}$ (the smallest detectable of the change,) the distance moved is $\Delta x \simeq 0.015\ \mathrm{mm}$.

Problem 1: Electrical Blackbox: Capacitive Displacement Sensor

Part 1. Calibration (3.0 Points)

Physical concepts/Understanding (1.3 Points) (Marks awarded: either full marks or zero)	
Points	**Concepts/Details**
0.4	**P1** Adding capacitance values by parallel configuration = check from values
0.4	**P2** At least one capacitance pair add up to be more than 151 pF
0.5	**P3** Plotting C and $1/f$ to form straight line graph or Plotting fC and f to form straight line graph Other graphs not allowed
Experimental skills and Analysis (1.2 Points)	
0.3	**E1** Measurements/data table of f and C (0.2). At least 2 correct units (0.1)
0.6	**E2** Graph: -> range of values along horizontal axis at least half a page (0.1) -> range of values along vertical axis at least half a page (0.1) -> correct plotting of data (0.2) -> horizontal axis units (0.1) -> vertical axis units (0.1)
0.3	**E3** Quality of data – number of data points: Options: at least 4 data points (0.3) or 3 or less (0)
Accuracy and uncertainties (0.5 Points)	
0.5	**A1** value of α 600 – 800 pF/ms (0.3) value of C_s 5 – 35 pF (0.2) Other values (0) Deduct 0.1 point if missing or incorrect unit Deduct 0.1 if more than 4 significant figures.

Part 2. Determination of geometrical shape of a parallel plate capacitor (6.0 Points)

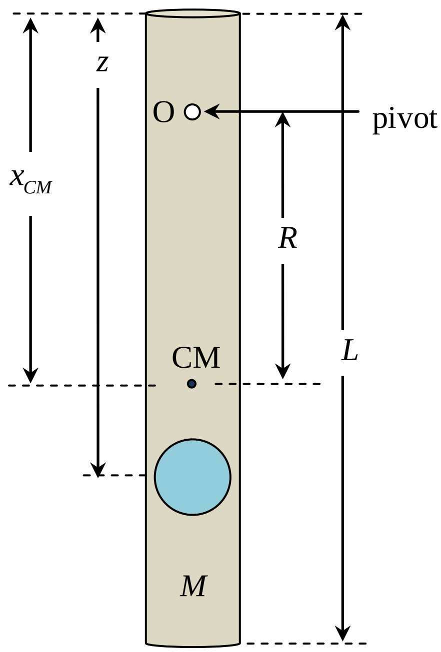
Points	**Concepts/Details**
Physical concepts/Understanding (1.4 Points) Drawing	
0.6	**P4** Plot of C versus distance (PATTERN I): -> Straight line up and down (0.3) -> Dropping/Increasing peaks on any of P4-P6 (0.2) -> Correct period of $2w$ (0.1) 
0.5	**P5**: Plot of C versus distance (PATTERN II) -> Options: curve with correct parabolic shape(0.2) or curve with cusp shape or like a Gaussian (0.1)  -> Correct period of $2w$ (0.1) -> Blank area – nearly flat/ slightly decreasing/ rounded. Successive blank areas can (but do not need to) change in level following the peaks (0.2).
0.3	**P6** Periods for PATTERN III -> Distance for non-blank area w (0.1) -> The overall period is $3w$ (0.2) 
Physical concepts/Understanding (1.5 Points) (Marks awarded: either full marks or zero)	
Points	**Concepts/Details**
0.5	**P7** Concept of parallel plate capacitor: $\frac{K\varepsilon_0 A}{d}$ (A can be replaced by formula for area)
0.5	**P8** Concept of using *the peaks* of C versus distance to find b
0.5	**P9** Concept of capacitance per sheet ΔC when varying the distance

Experimental skills and Analysis (2.6 Points)	
0.6	**E4** Table of data of x, f and C (0.4) units (0.2). Deduct 0.1 for each wrong or missing unit
0.6	**E5** Graph: -> range of values along horizontal axis at least half a page (0.1) -> range of values along vertical axis at least half a page (0.1) -> correct plotting of data (0.2) -> horizontal axis units (0.1) -> vertical axis units (0.1)
0.9	**E6** Quality of data – number of peaks: Options: 5 peaks or more (0.5), 3-4 peaks (0.3), 0-2 peaks (0) Plotting resolution: Options: about 1 mm (0.4), 2 mm (0.2), greater than 2.5 mm (0)
0.5	**E7** Find ΔC Options: use only difference between two peaks (0.1) use difference between the first and last peaks (0.3) average from at least 3 peaks (0.3) find a slope from at least 4 peaks (0.5) Use the same marking scheme if they do not use the peaks (e.g. they can use the troughs instead although this would give the wrong answer)
Accuracy and uncertainties (0.5 Points)	
0.3	**A2** value of w Options: 4.90 – 5.10 mm (0.3), other values (0) Deduct 0.1 point if missing or incorrect unit Deduct 0.1 point if more than 3 significant figures
0.2	**A2** value of b Options: 50 – 80 mm (0.2), other values (0) Deduct 0.1 point if missing or incorrect unit Deduct 0.1 point if more than 3 significant figures

Part 3. Resolution of digital calipers (1.0 Point)

Physical concepts/Understanding (0.4 Points)	
Points	**Concepts/Details**
0.3	**P10** Understand linearity of C with distance
0.1	**P11** $\Delta f = 0.01$ kHz to 0.05 kHz
Experimental skills and Analysis (0.3 Points)	
0.3	**E8** Find a slope of one section of the graph C vs. distance or f vs. distance.
Accuracy and uncertainties (0.3 Points)	
0.3	**A3** value of Δx Options: (1.5-1.8 mm/kHz)Δf (0.3) (1.0-1.4 mm/kHz)Δf or (1.9-2.2 mm/kHz)Δf (0.1) other values (0) Deduct 0.1 point if wrong or missing unit Deduct 0.1 point if more than 3 significant figures

Solution: 2 . Mechanical Blackbox: a cylinder with a ball inside



In order to be able to calculate the required values in i, ii, iii, we need to know:

a. the position of the centre of mass of the tubing plus particle (object) which depends on z, m, M
b. the moment of inertia of the above.

The position of the CM may be found by balancing. The I_{CM} can be calculated from the period of oscillation of the tubing plus object.

Analytical steps to select parameters for plotting

I. $$x_{CM} = \frac{mz + M\frac{L}{2}}{m+M} \qquad \text{……………………… (1)}$$

L is readily obtainable with a ruler.

x_{CM} is determined by balancing the tubing and object.

II. For small-amplitude oscillation about any point O the period T is given by considering the equation:

$$\left\{(M+m)R^2+I_{CM}\right\}\ddot{\theta} = -g(M+m)R\sin\theta \approx -g(M+m)R\theta \qquad \text{.......... (2)}$$

$$T = 2\pi\sqrt{\frac{I_{CM}+(M+m)R^2}{g(M+m)R}} \qquad \text{.......... (3)}$$

where
$$I_{CM} = \frac{1}{3}M\left(\frac{L}{2}\right)^2+M\left(x_{CM}-\frac{L}{2}\right)^2+m\left(z-x_{CM}\right)^2$$
$$= \frac{1}{3}ML^2+Mx_{CM}^2-MLx_{CM}+m\left(z-x_{CM}\right)^2 \qquad \text{.......... (4)}$$

Note that

$$T^2\frac{g(M+m)}{4\pi^2} = \frac{I_{CM}}{R}+(M+m)R \qquad \text{.......... (5)}$$

Method (a): (linear graph method)

The equation (5) may be put in the form:

$$T^2R = \left(\frac{4\pi^2}{g}\right)R^2+\frac{4\pi^2 I_{CM}}{(M+m)g} \qquad \text{.......... (6)}$$

Hence the plot of T^2R v.s. R^2 will yield the straight line whose

Slope $\alpha = \dfrac{4\pi^2}{g}$ .... (7)

and y-intercept $\beta = \dfrac{4\pi^2 I_{CM}}{(M+m)g}$ .... (8)

Hence, $I_{CM} = (M+m)\dfrac{\beta}{\alpha}$ .... (9)

The value of g is from equation (7): $g = \dfrac{4\pi^2}{\alpha}$ .... (10)

Method (b): minimum point curve method

The equation (5) implies that T has a minimum value at

$$R = R_{\min} \equiv \sqrt{\frac{I_{CM}}{M+m}} \qquad \text{……………………. (11)}$$

Hence $R_{\min}$ can be obtained from the graph T v.s. R.

And therefore

$$I_{CM} = (M+m)R_{\min}^2 \qquad \text{……………………. (12)}$$

This equation (12) together with equation (1) will allow us to calculate the required values z and $\frac{M}{m}$.

At the value $R = R_{\min}$ equation (5) becomes $T_{\min}^2 \frac{g(M+m)}{4\pi^2} = (M+m)R_{\min} + (M+m)R_{\min}$

$$g = \frac{2R_{\min}}{T_{\min}^2} \times 4\pi^2 = \frac{8\pi^2 R_{\min}}{T_{\min}^2} \qquad \text{……………………. (13)}$$

from which g can be calculated.

Results

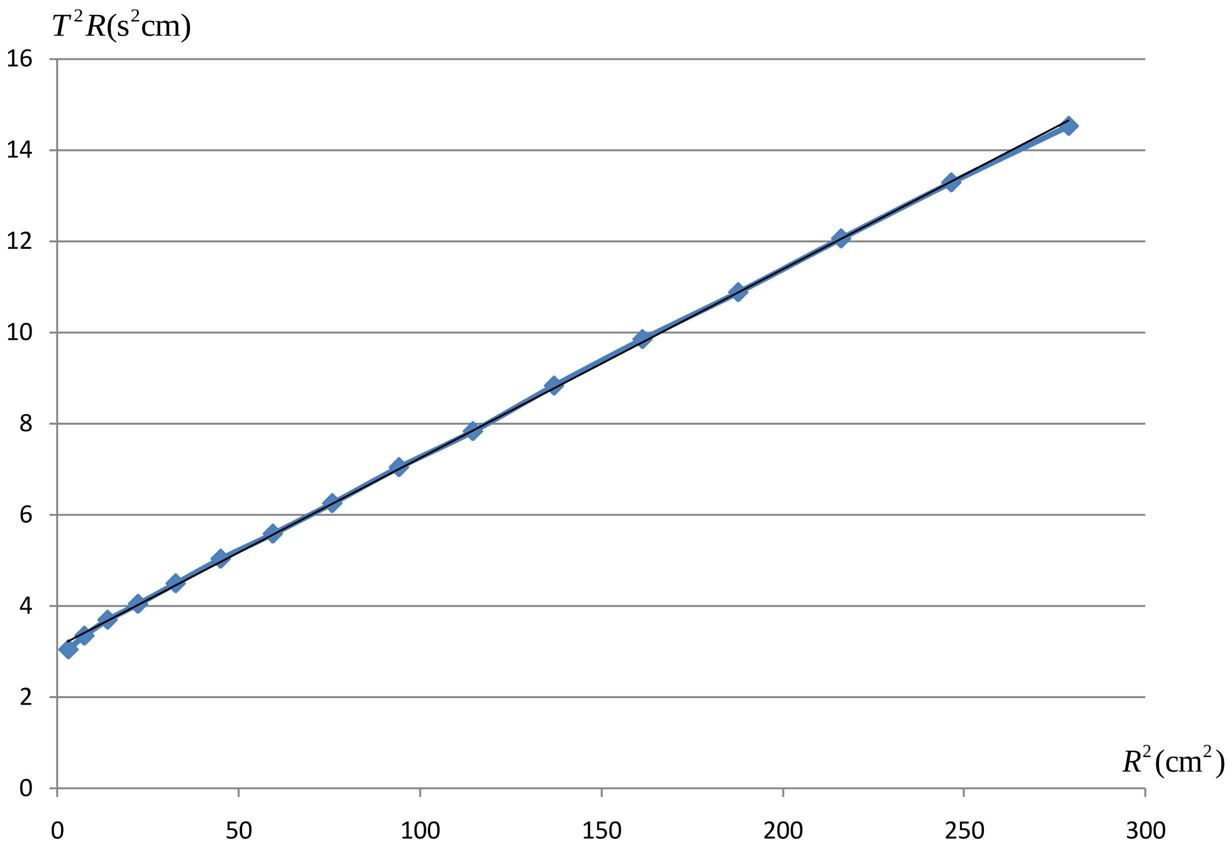
$$L = 30.0 \text{ cm} \pm 0.1 \text{ cm}$$

$$x_{CM} = 17.8 \text{ cm} \pm 0.1 \text{ cm (from top)}$$

$x_{CM} - R$ (cm)	time (s) for 20 cycles			T (s)	R (cm)	R^2 (cm^2)	T^2R (s^{2}cm)
1.1	18.59	18.78	18.59	0.933	16.7	278.9	14.53
2.1	18.44	18.25	18.53	0.920	15.7	246.5	13.29
3.1	18.10	18.09	18.15	0.906	14.7	216.1	12.06
4.1	17.88	17.78	17.81	0.891	13.7	187.7	10.88
5.1	17.69	17.50	17.65	0.881	12.7	161.3	9.85
6.1	17.47	17.38	17.28	0.869	11.7	136.9	8.83
7.1	17.06	17.06	17.22	0.856	10.7	114.5	7.83
8.1	17.06	17.00	17.06	0.852	9.7	94.1	7.04
9.1	16.97	16.91	16.96	0.847	8.7	75.7	6.25
10.1	17.00	17.03	17.06	0.852	7.7	59.3	5.58
11.1	17.22	17.37	17.38	0.866	6.7	44.9	5.03
12.1	17.78	17.72	17.75	0.888	5.7	32.5	4.49
13.1	18.57	18.59	18.47	0.927	4.7	22.1	4.04
14.1	19.78	19.90	19.75	0.991	3.7	13.7	3.69
15.1	11.16	11.13	11.13	1.114	2.7	7.3	3.34
16.1	13.25	13.40	13.50	1.338	1.7	2.9	3.04

Notes: at $x_{CM} - R = 15.1, 16.1$ cm, times for 10 cycles.

Method (a)



Calculation from straight line graph: slope $\alpha = 0.04108 \pm 0.0007$ s^2/cm, y-intercept $\beta = 3.10 \pm 0.05$ s^{2}cm

$$g = \frac{4\pi^2}{\alpha} \text{ giving } g = (961 \pm 20) \text{ cm/s}^2$$

$$\frac{\beta}{\alpha} = \frac{3.10}{0.04108} = 75.46 \text{ cm}^2 \left(\pm 2.5\,\text{cm}^2\right)$$

$$I_{CM} = \left(M+m\right)\frac{\beta}{\alpha} = \left(75.46\right)\left(M+m\right)$$

From equation (4): $I_{CM} = \frac{1}{3}M\left(\frac{L}{2}\right)^2 + M\left(x_{CM} - \frac{L}{2}\right)^2 + m\left(z - x_{CM}\right)^2$

Then $$(75.46)(M+m) = 75.0M + 7.84M + m(z-17.8)^2$$

$$-7.38\frac{M}{m} + 75.46 = (z-17.8)^2 \qquad \text{……………………. (14)}$$

The centre of mass position gives:

$$17.8(M+m) = 15.0M + mz$$

$$\frac{M}{m} = \frac{z-17.8}{2.8} \qquad \text{……………………. (15)}$$

From equations (14) and (15):

$$-\frac{7.38}{2.8}(z-17.8) + 75.46 = (z-17.8)^2$$

$$(z-17.8) = 7.47$$

And $$z = 25.27 = 25.3 \pm 0.1 \text{ cm}$$

$$\frac{M}{m} = 2.68 = 2.7$$

Error Estimation

Find error for g:

From (10), $$g = \frac{4\pi^2}{\alpha}$$

$$\Delta g = \frac{\Delta\alpha}{\alpha} g = 16.3 \text{ cm/s}^2 \approx 20 \text{ cm/s}^2$$

i) Find error for z:

First, find error for $r = \frac{\beta}{\alpha} = \frac{3.10}{0.04108} = 75.46 \text{ cm}^2$.

$$\Delta r = \left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\beta}{\beta}\right) r = 2.5 \text{ cm}^2$$

Since error from r contributes most ($\frac{\Delta r}{r} \sim 0.03$ while $\frac{\Delta L}{L}, \frac{\Delta x_{cm}}{x_{cm}} \sim 0.005$), we estimate error propagation from r only to simplify the analysis by substituting the min and max values into equation (4).

Now, we use $r_{\max} = r + \Delta r = 75.46 + 2.5 = 77.96$. The corresponding quadratic equation is $(z-17.8)^2 + 1.743(z-17.8) - 77.96 = 0$ The corresponding solution is $(z-17.8)_{\max} = 7.55$ cm

If we use $r_{\min} = r - \Delta r = 75.46 - 2.5 = 72.96$, the corresponding quadratic equation is

$$(z-17.8)^2 + 3.529(z-17.8) - 72.96 = 0$$

The corresponding solution is $(z-17.8)_{\min} = 6.96$ cm

So $\Delta(z-17.8) = \dfrac{7.55-6.96}{2} = 0.3$ cm

Note that $\dfrac{\Delta(z-17.8)}{z-17.8} \sim 0.04$. So, we still ignore the error propagation due to $\Delta L, \Delta x_{cm}$

The error Δz can be estimated from $\Delta z \approx \Delta(z-17.8) = 0.3$ cm

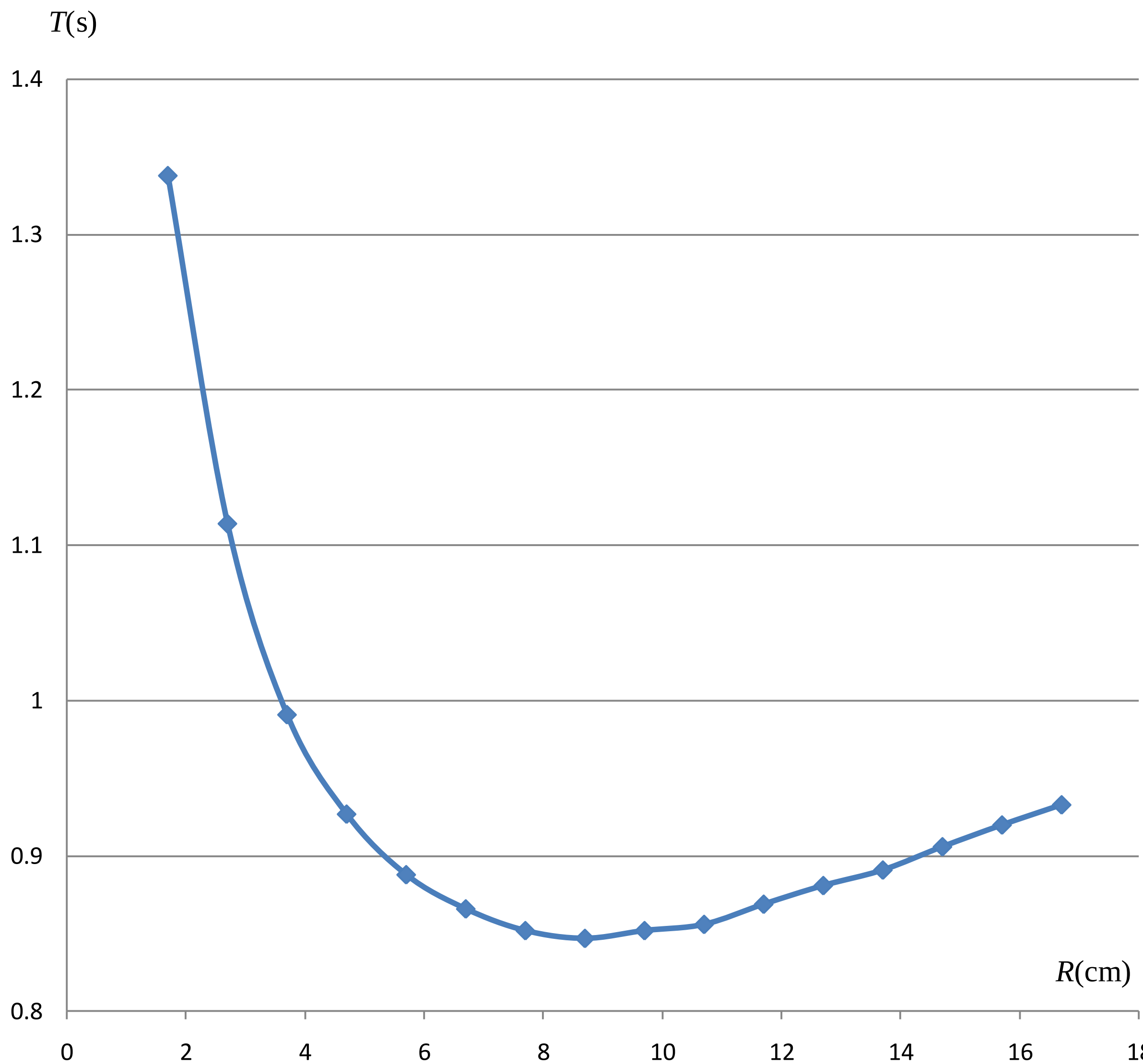
ii) Find error for $\dfrac{M}{m}$:

We know that $\dfrac{M}{m} = \dfrac{z-17.8}{2.8}$

$$\Delta\left(\frac{M}{m}\right) = \frac{\Delta(z-17.8)}{2.8} = 0.11$$

Method (b)

Calculation from *T*-*R* plot:



Using the minimum position: $T = T_{\min}$ at $I_{CM} = (M+m)R_{\min}^2$ and $g = \dfrac{8\pi^2 R_{\min}}{T_{\min}^2}$

From graph: $R_{\min} = 8.9 \pm 0.2$ cm and $T_{\min} = 0.846 \pm 0.005$ s

$$\therefore \quad g = 982 \pm 40 \text{ cm/s}^2$$

$$I_{CM} = (M+m)(8.9)^2 = (79.21)(M+m) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

From equations (14), (15), (16):

$$(79.21)(M+m) = 75.0M + 7.84M + m(z-17.8)^2$$

$$-3.63M + 79.21m = m(z-17.8)^2$$

$$(x-17.8)^2 + \frac{3.63}{2.8}(x-17.8) - 79.21 = 0$$

$$(z-17.8) = 8.28$$

And $z = 26.08 = 26.1 \pm 0.7$ cm

$$\frac{M}{m} = 2.95 = 3.0 \pm 0.3$$

Error estimation

i) Find error for g:

Using the minimum position: $g = \dfrac{8\pi^2 R_{\min}}{T_{\min}^2}$, we have

$$\Delta g = \left(\frac{\Delta R_{\min}}{R_{\min}} + 2\frac{\Delta T_{\min}}{T_{\min}}\right) g = 34 \approx 30 \text{ cm/s}^2$$

ii) Find error for z:

First, find error for $r = R_{\min}^2 = 79.21 \text{ cm}^2$.

$$\Delta r = 2R_{\min}\Delta R_{\min} = 3.56 \text{ cm}^2$$

This r is equivalent to r in part 1. So, one can follow the same error analysis.
As a result, we have

$$z = 26.08 \approx 26.1 \text{ cm}$$
$$\Delta z = 0.8 \text{ cm}$$

i) Find error for $\dfrac{M}{m}$:

Following the same analysis as in part I, we found that

$$\frac{M}{m} = 2.96 ; \Delta(\frac{M}{m}) = 0.15$$

NOTE: This minimum curve method is not as accurate as the usual straight line graph.

Problem 2: Mechanical Blackbox

I. Determination of CM (1.0 points) *marks are either full or zero

Physical concepts/Understanding (0.4 points)	
Points	**Concepts/Details**
0.4	**P1*** Method for CM measurement (schematic drawing) is scientifically reasonable: e.g. hanging the cylinder with a thread loop, hanging with strings at ends, placing at edge of table or moving balance points together until they meet.
Experimental skills and Analysis (0.2 points)	
0.2	**E1*** >=3 measurements
Accuracy and uncertainties (0.4 points) (penalty for unsuitable sig. figs. (-0.1) and missing units (-0.1))	
0.2	**A1*** Position of centre of mass 17.6 – 18.0 cm (from light end), 12.0-12.4 cm (from heavy end)
0.2	**A2** Error estimate $\leq 0.3\,\text{cm}$ from statistical error (0.2), 0.1-0.2 cm from single measurement error (0.1)

II: Determination of other parameters (9.0 points) *marks are either full or zero

Points	**Concepts/Details**
Physical concepts/Understanding (2.2 points)	
0.4	**P2*** Obtain expression for the period/frequency: e.g. using formula for simple harmonic motion, solving differential equation etc.
1.0	**P3*** Form a straight line equation that leads to a graph (e.g. T^2R vs. R^2 or T^2/R vs. $1/R^2$) to extract relevant parameters.
0.4	**P4*** $I_{CM} = \frac{1}{3}M\left(\frac{L}{2}\right)^2 + M\left(x_{CM} - \frac{L}{2}\right)^2 + m\left(z - x_{CM}\right)^2$
0.4	**P5*** $x_{CM} = \dfrac{mz + M\frac{L}{2}}{m+M}$
Experimental skills and Data analysis (3.7 points)	
0.6	**E2** Table: measurements T (0.2), R (0.2) and units (0.2)
1.0	**E3** Graph: appropriate scale to cover good area of the graph paper (area enclosing data points plotted covers at least half of graph paper area) (0.3)*, correct plotting of data (all correct (0.4)/some incorrect (0.2)/all wrong (0)) and units (0.3)

Points	Concepts/Details
1.3	**E4** Quality of data: For each measurement: >=10 oscillations (0.5), >=7 oscillations (0.3), others (0) -Number of measurement at each pivoting position: >=3 meas. (0.3), 2 meas. (0.1) , 1 meas. (0 pt) -Number of pivoting positions: >= 10 pos. (0.5), >= 8 pos. (0.4) , >= 5 pos. (0.3), < 5 (0).
0.4	**E5** Form two equations between z and M/m. (0.2 each)
0.4	**E6** Use these equations to find z (0.2) and M/m (0.2).
Accuracy and uncertainties (3.1 points) **(penalty for unsuitable sig. figs. (-0.1) and missing units (-0.1))**	
0.6	**A3** Obtain a correct value of g from the slope of the graph. The value of g 968 – 988 (0.6) 958 – 967 or 989 – 998 (0.3) cm/s^2
0.3	**A4** Equation for finding error of g (0.2), acceptable method of finding the precursor error(s) (0.1).
0.6	**A5** Obtain a correct value of z The value of z 25.9 – 26.2 (0.6) 25.5 – 25.8 or 26.3 – 26.6 (0.3) cm
0.6	**A6** Obtain a correct value of M/m The value of M/m 2.6 – 2.8 (0.6) 2.5 – 2.59 or 2.81 – 2.9 (0.3)
0.6	**A7** Equation for finding error of z (0.2), acceptable method of finding the precursor error(s) (0.1). Equation for finding error of (M/m) (0.2), acceptable method of finding the precursor error(s) (0.1).
0.4	**A8*** $\Delta z \le 0.4\,\text{cm}$ or $\Delta(M/m) \le 0.15$