Solution

Task 1

1.

1.1. $T_0 =$ 25±1 ^{o}C

$$V_{\text{samp}}\left(T_0\right) = 573.9 \text{ mV}$$

With different experiment sets, V_{samp} may differ from the above value within ±40 mV.

Note for error estimation:
δV and δV are calculated using the specs of the multimeter: ±0.5% reading digit +2 on the last digit. Example: if V = 500mV, the error δV = 500×0.5% + 0.2 = 2.7 mV ≈ 3 mV.

Thus, $V_{samp}\left(T_0\right) = 574 \pm 3 \text{ mV}$.

All values of $V_{samp}\left(T_0\right)$ within 505÷585 mV are acceptable.

1.2. Formula for temperature calculation:

From Eq (1): $V_{\text{samp}} = V_{\text{samp}}(T_0) - \alpha(T - T_0)$

$$V_{\text{samp}}\left(50^{o}\text{C}\right) = 523.9 \text{ mV}$$

$$V_{\text{samp}}\left(70^{o}\text{C}\right) = 483.9 \text{ mV}$$

$$V_{\text{samp}}\left(80^{o}\text{C}\right) = 463.9 \text{ mV}$$

Error calculation: $\delta V_{\text{samp}} = \delta V_{\text{samp}}\left(T_0\right) + \left(T - T_0\right)\delta\alpha$

Example: $V_{\text{samp}} = 495.2 \text{ mV}$, then $\delta V_{\text{samp}} = 2.7 + 0.03 \times (50 - 25) = 3.45 \text{ mV} \approx 3.5 \text{ mV}$

Thus:

$$V_{\text{samp}}\left(50^{o}\text{C}\right) = 524 \pm 4 \text{ mV}$$

$$V_{\text{samp}}\left(70^{o}\text{C}\right) = 484 \pm 4 \text{ mV}$$

$$V_{samp}\left(80^{o}\mathrm{C}\right) = 464 \pm 5 \text{ mV}$$

The same rule for acceptable range of V_{samp} as in 1.1 is applied.

2.

2.1. Data of cooling-down process without sample:

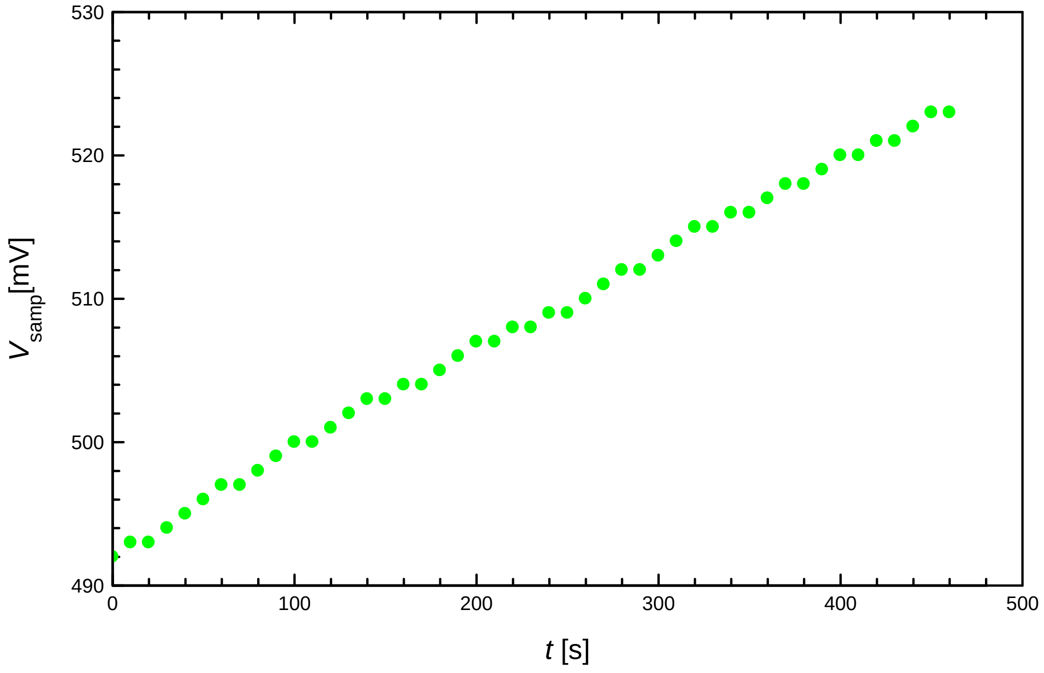
t (s)	V_{samp} (mV) (±3mV)	ΔV (mV) (±0.2mV)
0	492	-0.4
10	493	-0.5
20	493	-0.5
30	494	-0.6
40	495	-0.7
50	496	-0.7
60	497	-0.8
70	497	-0.8
80	498	-0.9
90	499	-1.0
100	500	-1.0
110	500	-1.1
120	501	-1.1
130	502	-1.2
140	503	-1.2
150	503	-1.3
160	504	-1.3
170	504	-1.4
180	505	-1.5
190	506	-1.6
200	507	-1.6
210	507	-1.7
220	508	-1.7
230	508	-1.8
240	509	-1.8
250	509	-1.8
260	510	-1.9
270	511	-1.9

280	512	-1.9
290	512	-2.0
300	513	-2.0
310	514	-2.1
320	515	-2.1
330	515	-2.1
340	516	-2.1
350	516	-2.2
360	517	-2.2
370	518	-2.3
380	518	-2.3
390	519	-2.3
400	520	-2.4
410	520	-2.4
420	521	-2.5
430	521	-2.5
440	522	-2.5
450	523	-2.6
460	523	-2.6

The acceptable range of ΔV is ± 40 mV. There is no fixed rule for the change in ΔV with T (this depends on the positions of the dishes on the plate, etc.)

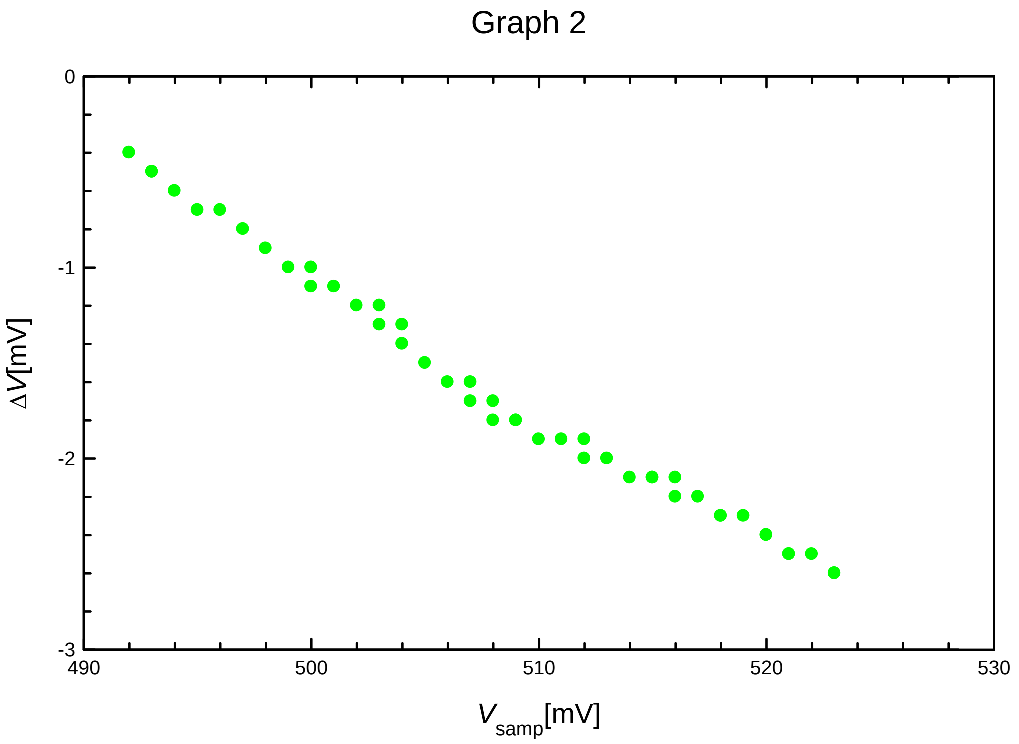
2.2.

Graph 1



The correct graph should not have any abrupt changes of the slope.

2.3.



The correct graph should not have any abrupt changes of the slope.

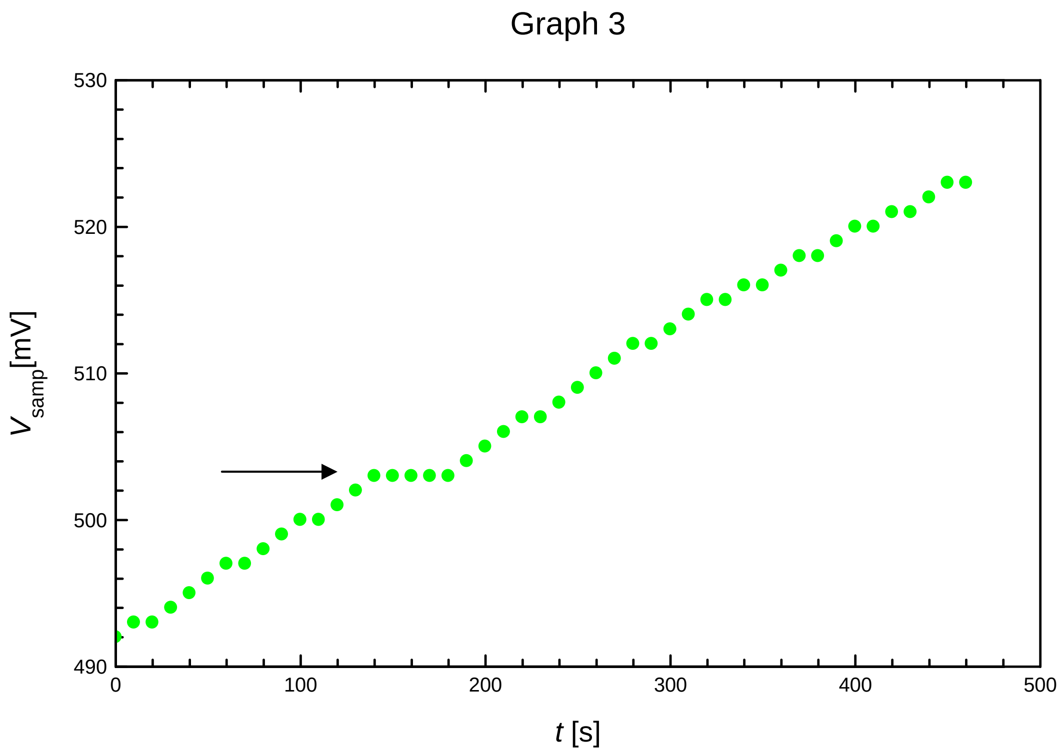
3.

3.1. Dish with substance

t (s)	V_{samp} (mV) (±3mV)	ΔV (mV) (±0.2mV)
0	492	-4.6
10	493	-4.6
20	493	-4.6
30	494	-4.6
40	495	-4.6
50	496	-4.6
60	497	-4.6
70	497	-4.5
80	498	-4.5
90	499	-4.5
100	500	-4.5
110	500	-4.5
120	501	-4.5

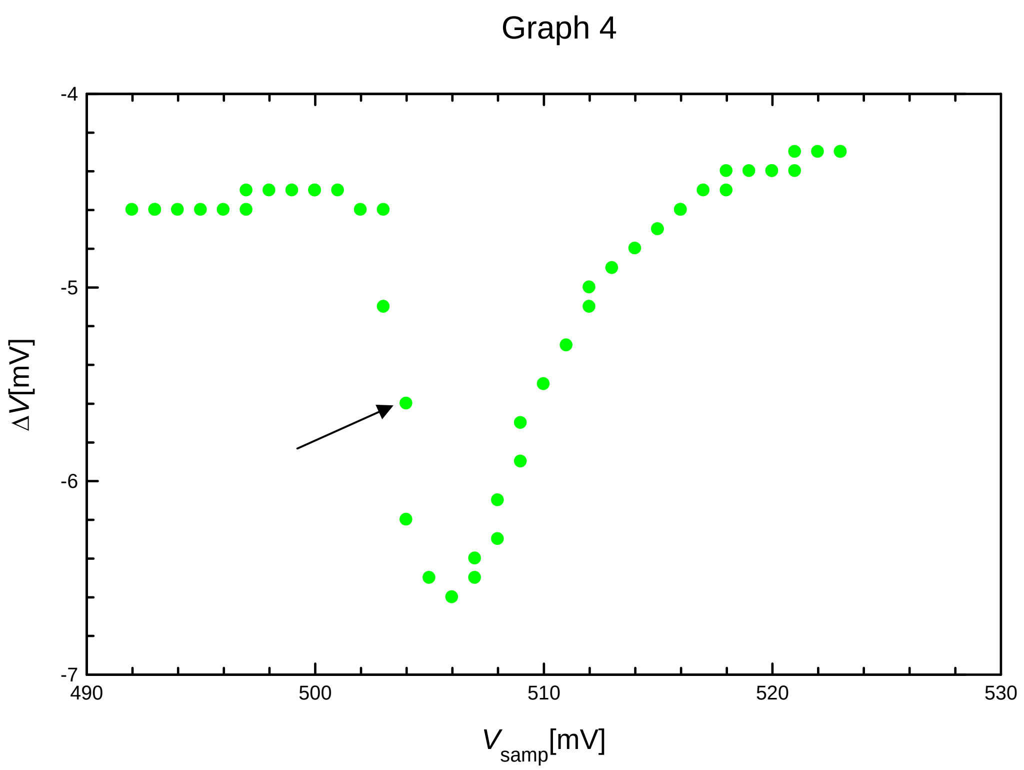
130	502	-4.6
140	503	-4.6
150	503	-5.1
160	503	-5.6
170	503	-6.2
180	503	-6.5
190	504	-6.6
200	505	-6.5
210	506	-6.4
220	507	-6.3
230	507	-6.1
240	508	-5.9
250	509	-5.7
260	510	-5.5
270	511	-5.3
280	512	-5.1
290	512	-5.0
300	513	-4.9
310	514	-4.8
320	515	-4.7
330	515	-4.7
340	516	-4.6
350	516	-4.6
360	517	-4.5
370	518	-4.5
380	518	-4.4
390	519	-4.4
400	520	-4.4
410	520	-4.4
420	521	-4.4
430	521	-4.3
440	522	-4.3
450	523	-4.3
460	523	-4.3

3.2.



The correct Graph 3 should contain a short plateau as marked by the arrow in the above figure.

3.3.



The correct Graph 4 should have an abrupt change in ΔV, as shown by the arrow in the above figure.

Note: when the dish contains the substance, values of ΔV may change compared to those without the substance.

4.

4.1. V_s is shown in Graph 3. Value V_s = (503±3) mV. From that, T_s = 60.5 ^oC can be deduced.

4.2. V_s is shown in Graph 4. Value V_s = (503±3) mV. From that, T_s = 60.5 ^oC can be deduced.

4.3. Error calculations, using root mean square method:

Error of T_s: $T_s = T_0 + \frac{V(T_0) - V(T_s)}{\alpha} = T_0 + A$, in which A is an intermediate variable.

Therefore error of T_s can be written as $\delta T_s = \sqrt{(\delta T_0)^2 + (\delta A)^2}$, in which δ... is the error.

Error for A is calculated separately:

$$\delta A = \frac{V(T_0) - V(T_s)}{\alpha}\sqrt{\left\{\frac{\delta[V(T_0) - V(T_s)]}{V(T_0) - V(T_s)}\right\}^2 + \left(\frac{\delta\alpha}{\alpha}\right)^2}$$

in which we have:

$$\delta[V(T_0) - V(T_s)] = \sqrt{[\delta V(T_0)]^2 + [\delta V(T_s)]^2}$$

Errors of other variables in this experiment:

$\delta T_0 = 1^oC$

$\delta V(T_0)$ = 3 mV, read on the multimeter.

$\delta\alpha$ = 0.03 mV/^oC

$\delta V(T_s) \approx$ 3 mV

From the above constituent errors we have:

$$\delta[V(T_0) - V(T_s)] \approx 4.24\, mV$$

$\delta A \approx 2.1°C$

Finally, the error of T_s is: $\delta T_s \approx 2.5°C$

Hence, the final result is: T_s =60±2.5 ^{o}C

Note: if the student uses any other reasonable error calculation method that leads to approximately the same result, it is also accepted.

Task 2

1.

1.1. $T_0 = 26 \pm 1^oC$

2.

2.1. Measured data with the lamp off

t (s)	$\Delta V(T_0)$ (mV) (±0.2mV)
0	19.0
10	19.0
20	19.0
30	19.0
40	19.0
50	18.9
60	18.9
70	18.9
80	18.9
90	18.9
100	19.0
110	19.0
120	19.0

Values of $\Delta V(T_0)$ can be different from one experiment set to another. The acceptable values lie in between -40÷+40 mV.

2.2. Measured data with the lamp on

t (s)	ΔV (mV) (±0.2mV)
0	19.5
10	21.9
20	23.8
30	25.5
40	26.9
50	28.0
60	29.0
70	29.9
80	30.7
90	31.4

100	32.0
110	32.4
120	32.9

When illuminated (by the lamp) values of ΔV may change 10 ÷ 20 mV compared to the initial situation (lamp off).

2.3. Measured data after turning the lamp off

t (s)	ΔV (mV) (±0.2mV)
0	23.2
10	22.4
20	21.6
30	21.0
40	20.5
50	20.1
60	19.6
70	19.3
80	18.9
90	18.6
100	18.4
110	18.2
120	17.9

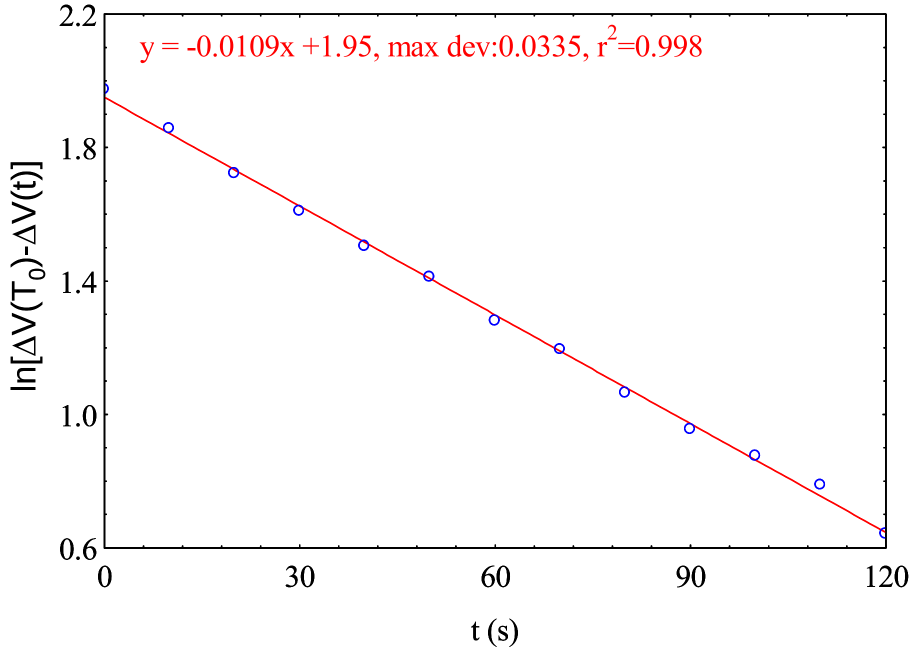
3. Plotting graph 5 and calculating k

3.1. $x = t$; $y = \mathbf{ln}\left[\Delta V\left(T_0\right) - \Delta V\left(t\right)\right]$

Note: other reasonable ways of writing expressions for x and y that also leads to a linear relationship using **ln** are also accepted.

3.2. Graph 5

Graph 5



3.3. Calculating k: $\frac{k}{C} =$ 0.0109 s^{-1} and $C = 0.69$ J/K, thus: $k = 7.52 \times 10^{-3}$ W/K

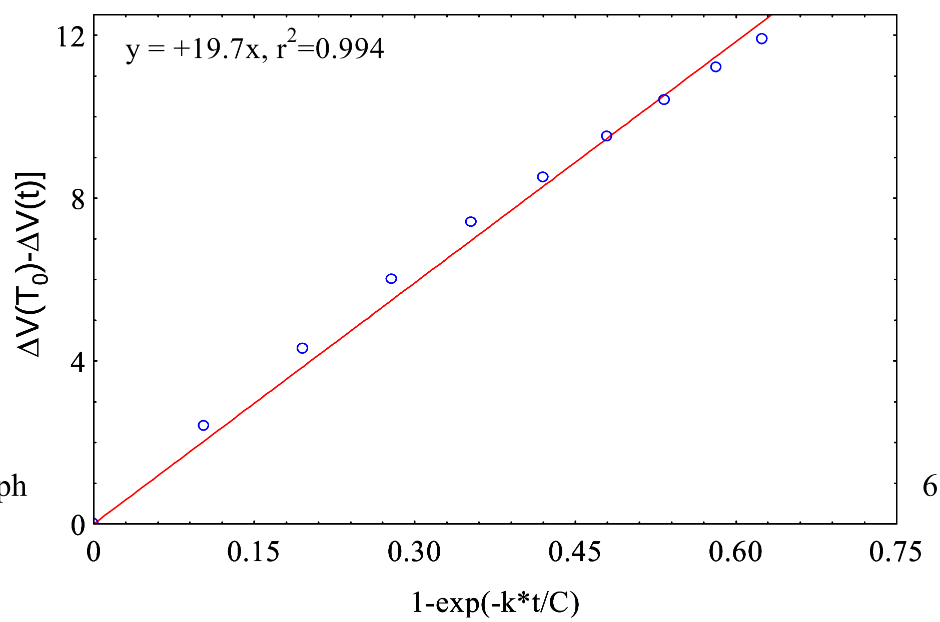
Note: Error of k will be calculated in 5.5. Students are not asked to give error of k in this step. The acceptable value of k lies in between $6\times10^{-3} \div 9\times10^{-3}$ W/K depending on the experiment set.

4. Plotting Graph 6 and calculating E

4.1. $x = \left[1 - \exp\left(\frac{-kt}{C}\right)\right]$; $y = \left|\Delta V(T_0) - \Delta V(t)\right|$

4.2.

Graph 6



Graph 6 should

be substantially linear, with the slope in between 15÷25 mV, depending on the experiment set.

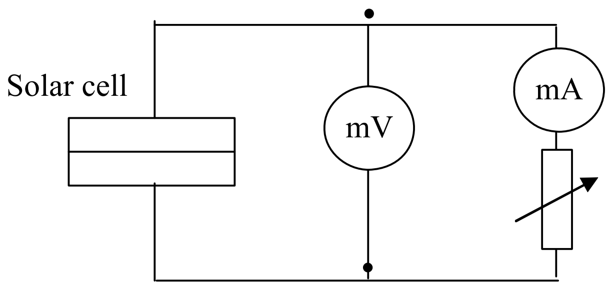
4.3. From the slope of Graph 6 and the area of the detector orifice we obtain E = 140 W/m^2. The area of the detector orifice is

$$S_{\text{det}} = \pi R_{\text{det}}{}^2 = \pi \times (13\times 10^{-3})^2 = 5.30\times 10^{-4}\ \text{m}^2 \quad \text{with error: } \frac{\delta R_{\text{det}}}{R_{\text{det}}} = 5\%$$

Error of E will be calculated in 5.5. Students are not asked to give error of E in this step. The acceptable value of E lies in between 120 ÷ 160 W/m^2, depending on the experiment set.

5.

5.1. Circuit diagram:



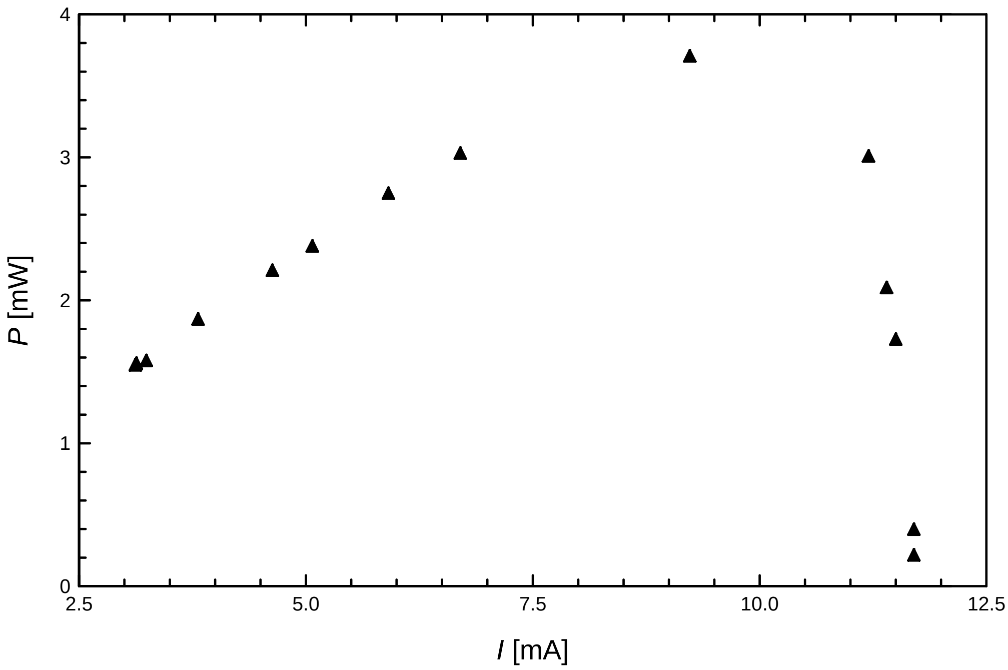
5.2. Measurements of V and I

V (mV) (±0.3÷3mV)	I (mA) (±0.05÷0.1mA)	P (mW)
18.6 ±0.3	11.7	0.21
33.5	11.7	0.39
150	11.5	1.72
157	11.6	1.82
182 ±1	11.4	2.08
267	11.2	3.00
402 ±2	9.23	3.70
448	6.70	3.02
459	5.91	2.74
468	5.07	2.37
473 ±3	4.63	2.20
480	3.81	1.86
485	3.24	1.57

487	3.12	1.54
489	3.13	1.55

5.3.

Graph 7



5.4. $P_{max} = 3.7 \pm 0.2$ mW

The acceptable value of P_{max} lies in between 3÷4.5 mW, depending on the experiment set.

5.5. Expression for the efficiency

$$S_{cell} = 19 \times 24\,\text{mm}^2 = 450 \times 10^{-6}\,\text{m}^2$$

Then $\eta_{max} = \dfrac{P_{max}}{E \times S_{cell}} = 0.058$

Error calculation:

$$\delta\eta_{max} = \eta_{max}\sqrt{\left(\frac{\delta P_{max}}{P_{max}}\right)^2 + \left(\frac{\delta E}{E}\right)^2 + \left(\frac{\delta S_{cell}}{S_{cell}}\right)^2}$$, in which S_{cell} is the area of the solar cell.

$\dfrac{\delta P_{max}}{P_{max}}$ is estimated from Graph 7, typical value ≈ 6 %

$\frac{\delta S_{\text{cell}}}{S_{\text{cell}}}$: error from the millimeter measurement (with the ruler), typical value ≈ 5 %

E is calculated from averaging the ratio (using Graph 6):

$$B = \frac{\Delta V(T_0) - \Delta V(t)}{1 - \exp\left(-\frac{k}{C}t\right)} = \frac{E\pi R_{\text{det}}{}^2\alpha}{k}$$

in which B is an intermediate variable, R_{det} is the radius of the detector orifice.

$$E = \frac{kB}{\pi R_{\mathbf{det}}{}^2\alpha}$$

Calculation of error of E:

$$\overline{\left(\frac{\delta E}{E}\right)} = \sqrt{\left(\frac{\delta k}{k}\right)^2 + \left(\frac{\delta B}{B}\right)^2 + 4\left(\frac{\delta R_{\mathbf{det}}}{R_{\mathbf{det}}}\right)^2 + \left(\frac{\delta\alpha}{\alpha}\right)^2}$$

k is calculated from the regression of:

$$\Delta T = \Delta T(0)\exp\left(-\frac{k}{C}t\right), \text{ hence } \ln\Delta T = \ln\Delta T(0) - \frac{k}{C}t$$

We set $k/C = m$ then $k = mC$

From the regression, we can calculate the error of m:

$$\frac{\delta m}{m} \approx 2(1-r) \approx 0.2\%$$

$$\frac{\delta k}{k} = \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta C}{C}\right)^2}$$

We derive the expression for the error of η_{max}:

$$\delta\eta_{\text{max}} = \eta_{\text{max}}\sqrt{\left(\frac{\delta P_{\text{max}}}{P_{\text{max}}}\right)^2 + \left(\frac{\delta S_{\text{cell}}}{S_{\text{cell}}}\right)^2 + \left(\frac{\delta B}{B}\right)^2 + 4\left(\frac{\delta R_{\mathbf{det}}}{R_{\mathbf{det}}}\right)^2 + \left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta C}{C}\right)^2 + \left(\frac{\delta\alpha}{\alpha}\right)^2}$$

Typical values for η_{max} and other constituent errors:

$$\eta_{\mathbf{max}} \approx 0.058$$

$$\frac{\delta P_{\mathbf{max}}}{P_{\mathbf{max}}} = 5\% \ ; \quad \frac{\delta B}{B} \approx 0.6\% \ ; \quad \frac{\delta m}{m} \approx 0.2\% \ ; \quad \frac{\delta S_{\text{cell}}}{S_{\text{cell}}} \approx 5\% \ ; \quad \frac{\delta R_{\mathbf{det}}}{R_{\mathbf{det}}} \approx 5\% \ ;$$

$$\frac{\delta C}{C} \approx 3\% ; \frac{\delta k}{k} \approx 3\% ; \quad \frac{\delta E}{E} \approx 10.5\% ; \quad \frac{\delta \alpha}{\alpha} \approx 1.5\%$$

Finally:

$$\frac{\delta \eta_{\max}}{\eta_{\max}} = 12.7\% ; \quad \delta \eta_{\max} \approx 0.0074$$

and

$$\eta_{\max} = \left(5.8 \pm 0.8\right)\%$$

Note: if the student uses any other reasonable error method that leads to approximately the same result, it is also accepted.