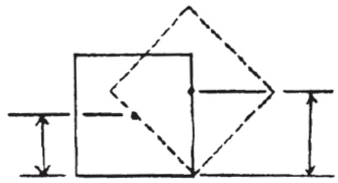
NATUURKUNDE OLYMPIADE 2015 Eindronde
ANTWOORDEN Theoretische toets



Opgave 1 Blokje

Conservation of Energy: If the block tips over about point *D*, it must at least achieve the dash position shown. Datum is set at point *D*. When the block is at its initial and final position, its center of gravity is located 0,5 m and 0,7071 m *above* the datum. Its initial and final potential energy are 9,81·10·0,5 = 49,05 J and 9,81·10·0,7071 = 69,37 J.
The mass moment of inertia of the block about point *D* is

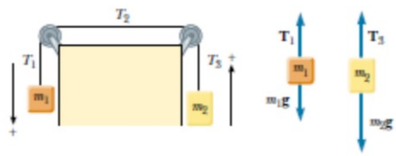
$$I_D = \frac{1}{12}10(1^1 + 1^1) + 10\left(\sqrt{0{,}5^2 + 0{,}5^2}\right)^2 = \frac{10}{12} + \frac{10}{2} = 5{,}83$$

The initial kinetic energy of the block (after the impact) is $\frac{1}{2}I_D\omega_2^2$.
We have:

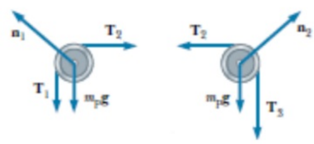
$$T_2 + V_2 = T_3 + V_3$$
$$\tfrac{1}{2}I_D\omega_2^2 + 49{,}05 = 0 + 69{,}37$$
$$\omega_2 = 2{,}64 \text{ rad/s}$$

Conservation of Angular Momentum: Since the weight of the block and the normal reaction *N* are *nonimpulsive* forces, the angular momentum is conserves about point *D*. We have

$$L_1 = L_2$$
$$10v \cdot 0{,}5 = I_D\omega_2$$
$$v = \frac{5{,}83 \cdot 2{,}64}{5} = 3{,}08 \text{ m/s}$$



Opgave 2 Atwood's machine revisited



We shall define the downward direction as positive for m_1 and upwards as the positive direction for m_2. This allows us to represent the acceleration of both masses by a single variable a ans also enables us to relate a positive a to a positive (counterclockwise) angular acceleration α. Let us write Newton's second law of motion for each block, using the free-body diagrams for the two blocks as shown in the figure:

$$m_1 g - T_1 = m_1 \qquad [1]$$
$$T_3 - m_2 g = m_2 a \qquad [2]$$

Next, we must include the effect of the pulleys on the motion. Free-body diagrams for the pulleys are shown in the figure. The net torque about the axle for the pulley on the left is $(T_1 - T_2)R$ while the net torque for the pulley on the right is $(T_2 - T_3)R$. Using the relation $\sum\tau = I\alpha$ for each pulley and noting that each pulley has the same angular acceleration α, we obtain

$$(T_1 - T_2)R = I\alpha \qquad [3]$$
$$(T_2 - T_3)R = I\alpha \qquad [4]$$

We now have four equations with four unknowns.
These can be solved simultaneously. Adding [3] and [4] gives

$$(T_1 - T_3)R = 2I\alpha \qquad [5]$$

Adding equations [1] and [2] gives

$$T_3 - T_1 + m_1 g - m_2 g = (m_1 + m_2)a$$
$$T_1 - T_3 = (m_1 - m_2)g - (m_1 + m_2)a \qquad [6]$$

Substituting equation [6] into [5], we have

$$[(m_1 - m_2)g - (m_1 + m_2)a]R = 2I\alpha$$

Because $\alpha = a/R$, this expression can be simplified to

$$(m_1 - m_2)g - (m_1 + m_2)a = 2I\frac{a}{R^2}$$

This can be rewritten:

$$g = a\frac{\left(m_1 + m_2 + \frac{2I}{R^2}\right)}{(m_1 - m_2)}$$

Opgave 3 (Niet) verbonden

Let us identify the left-hand plates of the capacitors as an isolated system because they are not connected to de right hand plates by conductors. The charges on the left hand plates before the switches are closed are:

$$Q_{1i} = C_1\Delta V_i \text{ and } Q_{2i} = -C_2\Delta V_i$$

The negative sign for Q_{2i} is necessary because the charge on the left plate of capacitor C_2 is negative. The total charge Q in the system is:

$$Q = Q_{1i} + Q_{2i} = (C_1 - C_2)\Delta V_i \qquad [1]$$

After the switches are closed, the total charge in the system remains the same:

$$Q = Q_{1f} + Q_{2f} \qquad [2]$$

The charge redistribute until the entire system is at the same potential ΔV_f. Thus, the final potential difference across C_1 must be the same as the final potential difference across C_2. To satisfy this requirement, the charges on the capacitors after the switches are close are:

$$Q_{1f} = C_1\Delta V_f \text{ and } Q_{2f} = -C_2\Delta V_f$$

Deviding the first equation by the second we have

$$\frac{Q_{1f}}{Q_{2f}} = \frac{C_1\Delta V_f}{C_2\Delta V_f} = \frac{C_1}{C_2}$$
$$Q_{1f} = \frac{C_1}{C_2}Q_{2f} \qquad [3]$$

Combining equations [2] and [3], we obtain

$$Q = Q_{1f} + Q_{2f} = \frac{C_1}{C_2}Q_{2f} + Q_{2f} = Q_{2f}\left(1 + \frac{C_1}{C_2}\right)$$
$$Q_{2f} = Q\left(\frac{C_1}{C_1 + C_2}\right)$$

Using equation [3] to fins Q_{1f}in terms of Q, we have:

$$Q_{1f} = \frac{C_1}{C_2}Q_{2f} = \frac{C_1}{C_2}Q\left(\frac{C_1}{C_1 + C_2}\right) = Q\left(\frac{C_1}{C_1 + C_2}\right)$$

Finally, we find for the voltage across each capacitor:

$$\Delta V_{1f} = \frac{Q_{1f}}{C_1} = \frac{Q\left(\frac{C_1}{C_1 + C_2}\right)}{C_1} = \frac{Q}{C_1 + C_2}$$

$$\Delta V_{2f} = \frac{Q_{2f}}{C_2} = \frac{Q\left(\frac{C_2}{C_1 + C_2}\right)}{C_2} = \frac{Q}{C_1 + C_2}$$

As noted earlier, $\Delta V_{1f} = \Delta V_{2f} = \Delta V_f$

To express ΔV_f in terms of the given quantities C_1, C_2 and ΔV_i we substitute the value of equation [1] to obtain:

$$\Delta V_f = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)\Delta V_i$$

Before the switches are closed, the total energy stored in the capacitors is:

$$U_i = \tfrac{1}{2}C_1(\Delta V_i)^2 + \frac{1}{2}C_2(\Delta V_i)^2 = \frac{1}{2}(C_1 + C_2)(\Delta V_i)^2$$

After the switches are closed, the total energy stored in the capacitors is

$$U_f = \tfrac{1}{2}C_1(\Delta V_f)^2 + \frac{1}{2}C_2(\Delta V_f)^2 = \frac{1}{2}(C_1 + C_2)(\Delta V_f)^2$$

$$U_f = \frac{1}{2}(C_1 + C_2)\left(\frac{Q}{C_1 + C_2}\right)^2 = \frac{1}{2}\frac{Q^2}{C_1 + C_2}$$

Using equation [1] we can express this as:

$$U_f = \frac{1}{2}\frac{Q^2}{C_1 + C_2} = \frac{1}{2}\frac{(C_1 - C_2)^2(\Delta V_i)^2}{(C_1 + C_2)}$$

Therefore, the ration of the final energy stored to the initial energy stored is:

$$\frac{U_f}{U_i} = \frac{\frac{1}{2}\frac{(C_1 - C_2)^2(\Delta V_i)^2}{(C_1 + C_2)}}{\frac{1}{2}(C_1 + C_2)(\Delta V_i)^2} = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2$$

Opgave 4 Spiegeltje

De hoeveelheid energie die de zon per seconde uitzendt is $P_{zon} = \sigma T_0^4$. Het deel dat in de spiegel terecht komt is $P_{zon} \cdot \frac{R^2}{D^2} = \sigma T_0^4 \frac{R^2}{D^2}$

Hierin is D de afstand van de zon tot de aarde. De zon wordt door de spiegel in het brandpunt afgebeeld met een lineaire verkleiningsfactor $\frac{L}{D}$. Het opgenomen vermogen door het plaatje is dan:

$$P_{in} = \sigma T_0^4 \frac{R^2}{D^2} \cdot \frac{D^2}{L^2} = \sigma T_0^4 \frac{R^2}{L^2}$$

Beschouw het - goed geïsoleerde - plaatje (evenals de zon) als een zwarte straler. Het plaatje kan de energie naar twee kanten uitstralen, zodat het uitgezonden vermogen is:

$$P_{uit} = 2\sigma T^4$$

In geval van evenwicht is $P_{in} = P_{uit}$.

Daaruit volgt voor de temperatuur van het plaatje: $T \approx 1600$ K.

Opgave 5 Ideaal gas

(a) Proces (1→2) is een irreversibel proces tegen een constante *buiten*druk, waarvoor geldt (met $pV = RT$ voor 1 mol ideaal gas):

$$W = -\int_{V_1}^{V_2} p^{ext} dV = -p^{ext}(2V_1 - V_1) = -\frac{p^{ext}}{p_1}RT_1$$

$W = -0{,}2 \times 8{,}314 \times 300 = -498{,}8\ \mathrm{J}$

Omdat de temperatuur aan het begin en eind van het proces hetzelfde is, en we in dit geval te maken hebben met een ideaal gas, is de energie toename gelijk aan nul (denk er aan: de energie is een toestandsgrootheid, warmte of arbeid niet!):

$$\Delta U = U_2 - U_1 = \frac{3}{2} R(T_2 - T_1) = 0$$

De toegevoerde warmte berekenen we met behulp van de eerste hoofdwet:

$$Q = \Delta U - W = -W = 498{,}8\ \mathrm{J}$$

Proces (2→3) is een isochore afkoeling, waarvoor geldt:

$$W = 0$$

$$Q = C_V \Delta T = \frac{3}{2} R(T_3 - T_2) = \frac{3}{2} \times 8{,}314 \times (250 - 300) = -623{,}6\ \mathrm{J}$$

$$\Delta U = Q + W == -623{,}6\ \mathrm{J}$$

Voor het complete proces vinden we dus: $W = -498{,}8\ \mathrm{J}$, $Q = -124{,}8\ \mathrm{J}$ en $\Delta U = -623{,}6\ \mathrm{J}$.

(b) De entropie toename van het systeem in proces (1→2) vinden we door een equivalent reversibel proces van dezelfde begintoestand naar dezelfde eindtoestand te construeren. Dit is een isotherme reversibele expansie van het ideale gas, waarvoor geldt ($n = 1\ \mathrm{mol}$):

$$\Delta S = \int \frac{q}{T} = \int \frac{p}{T} dV = \int \frac{R}{V} dV = R \ln \frac{V_2}{V_1}$$

$$\Delta S = R \ln 2 = 8{,}314 \times \ln 2 = 5{,}76 \frac{\mathrm{J}}{\mathrm{K}}$$

Waar we gebruik hebben gemaakt van $dU = q - pdV = 0$ (Pas op: dit geldt alleen in dit geval voor een ideaal gas bij constante temperatuur!) Voor proces (2→3) beschouwen we een reversibel isochore afkoeling, waarvoor geldt:

$$\Delta S = \int \frac{C_V dT}{T} = C_V \ln \frac{T_3}{T_2} = \frac{3}{2} \times 8{,}314 \times \ln \frac{250}{300} = -2{,}27\ \mathrm{J/K}$$

De totale entropie toename is dus $\Delta S = 3{,}49\ \mathrm{J/K}$

Opgave 6 *N* spleten

(a) Voor de irradiantie op het scherm onder een hoek $\theta = \frac{y}{L}$ geldt $I(\theta) = I_0 (\frac{\sin\beta}{\beta})^2 (\frac{\sin N\alpha}{\sin\alpha})^2$, met $\alpha = \frac{\pi a}{\lambda} \sin\theta$ en $\beta = \frac{\pi b}{\lambda} \sin\theta$. Er zijn 5 secundaire maxima tussen twee primaire maxima, Het aantal spleten is daardoor 7.

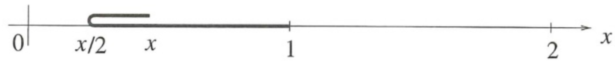
(b) Het eerste primaire maximum ligt op $y = 1{,}5$ cm boven het hoofdmaximum. Dan geldt dat $\sin\theta = \frac{y}{L} = \frac{\lambda}{a}$, hieruit volgt $a = \lambda \frac{l}{y} = 628 \frac{100}{1{,}5} = 41{,}9\ \mu\mathrm{m}$.

(c) $I(0) = 1{,}8 = I_o \cdot N^2$ en $I(y_1) = 1{,}575 = I_0 \cdot N^2 \left(\frac{\sin\beta_1}{\beta_1}\right)^2$.

Daaruit volgt: $\left(\frac{\sin\beta_1}{\beta_1}\right)^2 = \frac{1{,}575}{1{,}8} = 0{,}875$ en $\sin\beta_1 = 0{,}935\beta_1$.

taylorreeks: $\sin\beta_1 \sim \beta_1 - \frac{1}{6}\beta_1^3 = 0{,}935\beta_1 \rightarrow\ \beta_1 = 0{,}622$.

Hieruit volgt dan: $b = \frac{\lambda\beta_1}{\pi \sin\left(\tan^{-1}\frac{y_1}{L}\right)} = \frac{0{,}628 \cdot 0{,}622}{\pi \cdot 0{,}015} = 8{,}29\ \mu\mathrm{m}$.



Opgave 7 Vloerkleedje

Let the position of the moving end of the carpet be x as shown in the figure. It follows that the other end of the moving part is at $x/2$, and hence that the coordinate of its centre of mass is $3x/4$. Although $\mathrm{d}x/\mathrm{d}t = 1$, the speed of the centre of mass of the moving part is only $\frac{3}{4}$!

The linear momentum of the moving part is $p = mv$, where $v = 1$ and m is increasing uniformly with time. The net force acting on the moving part is thus

$$F = \frac{dp}{dt} = \frac{dm}{dt}v + \frac{dv}{dt}m = \frac{dm}{dt}1 + 0$$

The rate of change of the mass of the moving part can be found with the help of the following argument. The moving end of the carpet starts from the origin and the whole carpet will be moving when it reaches $x = 2$; this it does after two units of time, i.e. $dm/dt = \frac{1}{2}$. The corresponding minimal force (neglecting all dissipative forces) is $F = \frac{1}{2}$.

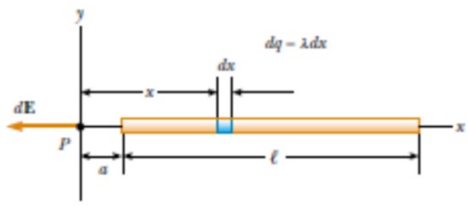
Notes.

(i) The centre of mass of the moving part of the carpet is initially at the origin and after two units of time at $x = \frac{3}{2}$, again showing that the speed of the centre of mass (v_{CM}) of the moving part is $\frac{3}{4}$.

(ii) Notice that the linear momentum of the moving part ($p = mv$) is *not* equal to the product of its mass and the speed of its centre of mass (mv_{CM}).

(iii) It seems tempting to try to find the minimum force required by using the conservation of energy, i.e. $F \times 2L = mv^2/2$, where L is the length of the carpet. The result would be $F = \frac{1}{4}$, which is only one-half of the value calculated earlier. The error in this argument is to ignore the continuous inelastic collisions which occur when the moving part of the carpet is jerking the next piece into motion. Half of the work goes into the kinetic energy of the carpet, but the other half is dissipated as heat.

Opgave 8 2X Geladen staaf



(a) Let us assume that the rod is lying along the x axis, that dx is the length of one small segment, and that dq is the charge on that segment. Because the rod has a charge per unit length λ, the charge dq on the small segment is $dq = \lambda dx$.

The field dE due to this segment at P is in the negative x direction (because the source of the field carries a positive charge Q), and its magnitude is

$$dE = k_e \frac{dq}{x^2} = k_e \lambda \frac{dx}{x^2}$$

Because every other element also produces a field in the negative x direction, the problem of summing their contributions is particularly simple in this case. The total field at P due to all segments of the rod, which are at different distances from P in this case becomes

$$E = \int_a^{l+a} k_e \lambda \frac{dx}{x^2}$$

Where the limits on the integral extend from one end of the rod ($x = a$) to the other ($x = l + a$). The constants k_e and λ can be removed from the integral to yield

$$E = k_e \lambda \int_a^{l+a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x}\right]_a^{l+a} = k_e \lambda \left(\frac{1}{a} - \frac{1}{l+a}\right) = \frac{k_e Q}{a(l+a)}$$

Where we have used the fact that the total charge $Q = \lambda l$.

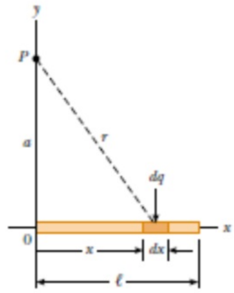
If P is far from the rod ($a \gg l$), then the l in the denominator can be neglected, and $E = k_e Q/a^2$. This is just the form you would expect for a point charge. Therefore, at large values of a/l, the charge distribution appears to be a point charge of magnitude Q. The use of the limiting technique ($a/l \rightarrow \infty$) often is a good method for checking a theoretical formula.

(b) The length element dx has a charge $dq = \lambda dx$. Because this element is a distance $r = \sqrt{x^2 + a^2}$ from point P, we can express the potential at point P due to this element as

$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda dx}{\sqrt{x^2 + a^2}}$$

To obtain the total potential at P, we integrate this expression over the limits $x = 0$ and $x = l$. Noting that k and λ are constants, we find that

$$V = k_e \lambda \int_0^l \frac{dx}{\sqrt{x^2 + a^2}} = k_e \frac{Q}{l} \int_0^l \frac{dx}{\sqrt{x^2 + a^2}}$$



This integral has the following value:

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Evaluating V, we find that

$$\int \frac{dx}{\sqrt{x^2 + a^2}} V = \frac{k_e Q}{l} \ln\left(\frac{l + \sqrt{l^2 + a^2}}{a}\right)$$

Opgave 9 2 kleine gaatjes

(a) Fase gat boven: $f_1 = \frac{2\pi d}{\lambda} \sin\phi + \frac{2\pi l_1}{\lambda}$

Fase gat onder: $f_2 = \frac{2\pi d}{\lambda} \sin\theta_m + \frac{2\pi l_1}{\lambda}$

Met l_1 de afstand tot punt op het scherm (voor gaten gelijk en onnodig voor oplossing…)

$f_1 - f_2 = \frac{2\pi d}{\lambda}(\sin\phi - \sin\theta_m) = m2\pi$ voor $m = 0, 1, 2, 3, \ldots$

Voor $m = 0$ het hoofdmaximum geldt dan:

$\sin\phi = \sin\theta_0 = \frac{y_0}{z} \rightarrow\ y_0 = z\sin\theta = 750 \sin 0{,}0004 = 0{,}3\ \text{mm}.$

(b) $f_1 = \frac{2\pi\delta}{\lambda} + \frac{2\pi d}{\lambda}\sin\phi + \frac{2\pi l_1}{\lambda}$

$f_2 = \frac{2\pi n\delta}{\lambda} + \frac{2\pi d}{\lambda}\sin\theta_m + \frac{2\pi l_1}{\lambda}$

$f_1 - f_2 = \frac{2\pi\delta}{\lambda} + \frac{2\pi d}{\lambda}\sin\phi - \frac{2\pi n\delta}{\lambda} - \frac{2\pi d}{\lambda}\sin\theta_m = \frac{2\pi\delta}{\lambda}(1-n) + \frac{2\pi d}{\lambda}(\sin\phi - \sin\theta_m) = m2\pi)$ voor een maximum.

Een maximum op $\theta = 0$ levert $\delta(1-n) + d\sin\phi = m\lambda \rightarrow\ \delta = \frac{d\sin\phi}{n-1} + m\frac{\lambda}{n-1}$

dus voor minimale δ geldt: $\delta_{min} = \frac{d\sin\phi}{n-1} = \frac{1\sin 0{,}0004}{1{,}5-1} = 0{,}8\ \mu\text{m}.$

(c) $\delta = \frac{d\sin\phi}{n-1} + 2m\lambda$

(d) $f_1 = \frac{2\pi\delta}{\lambda} + \frac{2\pi d}{\lambda}\sin\phi + \frac{2\pi l_1}{\lambda}$

$f_2 = \frac{2\pi n\delta}{\lambda} + \frac{2\pi d}{\lambda}\sin\theta_m + \frac{2\pi l_1}{\lambda}$

$f_1 - f_2 = \frac{2\pi\delta}{\lambda}(1-n) + \frac{2\pi d}{\lambda}(\sin\phi - \sin\theta_m) = \left(m + \frac{1}{2}\right)2\pi$ voor minimum.

Voor optische as geldt weer: $\theta = 0$: $\delta(1-n) + d\sin\phi = \left(m + \frac{1}{2}\right)\lambda$

$\delta = \frac{d\sin\phi}{n-1} + \left(m + \frac{1}{2}\right)\frac{\lambda}{n-1} = \delta_{min} + (2m+1)\lambda$ voor $n = 1{,}5$.

Opgave 10 Vierkantje

(a) Een manier om dit aan te pakken is door eerst de fluxtoename in de situatie te nemen waarin alleen de rechterkant van de winding naar rechts gaat, dan de fluxafname in de situatie waarin alleen de linkerkant van de winding naar rechts gaat en deze dan samen te nemen.

$d\Phi_1 = B_{x+a}dA$ voor rechts en $d\Phi_2 = -B_x dA$.

Voor het geheel geldt dan: $d\Phi = (B_{x+a} - B_x)dA = (B_{x+a} - B_x)adx$

Voor de inductiespanning moeten we dan schrijven:

$$U = \frac{d\Phi}{dt} = \frac{\mu_0 Ia}{2\pi}\left(\frac{1}{x+a} - \frac{1}{x}\right)v = \frac{\mu_0 Ia}{2\pi}\frac{a}{x(x+a)}v = \frac{\mu_0 Ia^2 v}{2\pi x(x+a)}$$

(b) Kwestie van zien hoe de afname in B in winding is gericht, die richting nemen en bepalen dat stroom daarvoor met de klok mee moet gaan.

Opgave 11 (Snelle) trein in een tunnel

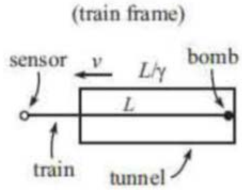


Figure 11.52

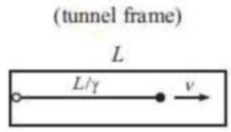


Figure 11.53

Yes, the bomb explodes. This is clear in the frame of the train (see Fig. 11.52). In this frame, the train has length L , and the tunnel speeds past it. The tunnel is length-contracted down to L/γ. Therefore the far end of the tunnel passes the front of the train before the near end passes the back, so the bomb explodes.

We can, however, also look at things in the frame of the tunnel see Fig. 11.53). Here the tunnel has length L, and the train is length-contracted down to L/γ. Therefore, the deactivation device gets triggered *before* the front of the train passes the far end of the tunnel, so you might think that the bomb does *not* explode. We appear to have a paradox.

The resolution to this paradox is that the deactivation device cannot instantaneously tell the bomb to deactivate itself. It takes a finite time for the signal to travel the length of the train from the sensor to the bomb. And it turns out that this transmission time makes it impossible for the deactivationsignal to get to the bomb before the bomb gets to the far end of the tunnel, no matter how fast the train is moving. Let's show this.

The signal has the best chance of winning this "race" if it has speed c, so let's assume this is the case. Now, the signal gets to the bomb before the bomb gets to the far end of the tunnel if and only if a light pulse emitted from the near end of the tunnel (at the instant the back of the train goes by) reaches the far end of the tunnel before the front of the train does. The former takes a time L/c. The latter takas a time $L(1-1/\gamma)/v$, because the front of the train is already a distance L/γ through the tunnel. So if the bomb is *not* to explode, we must have:

$$\frac{L}{c} < L(1-1/\gamma)/v$$

$$\leftrightarrow \beta < 1 - \sqrt{1-\beta^2}$$

$$\leftrightarrow \sqrt{1-\beta^2} < 1 - \beta$$

$$\leftrightarrow \sqrt{1+\beta} < \sqrt{1-\beta}$$

This is never true. Therefore, the signal always arrives too late, and the bomb always explodes.

Opgave 12 Roostermodel

(a) Plaats het eerste deeltje: er zijn M mogelijkheden. Voor het 2e deeltje zijn er nog $(M-1)$ mogelijkheden, enzovoorts. Dit leidt tot:

$$\Omega = M(M-1)(M-2)\dots\big(M-(N-1)\big)$$

Echter, omdat de deeltjes identiek zijn, hebben we te veel mogelijkheden geteld, want we hadden ook eerst deeltje 2 neer kunnen zetten. We moeten delen door alle mogelijke permutaties, dat is $N!$, dus:

$$\Omega = \frac{M(M-1)(M-2)\dots\big(M-(N-1)\big)}{N!}$$

$$\Omega = \frac{M(M-1)(M-2)\dots\big(M-(N-1)\big)\times(M-N)\big(M-(N+1)\big)\dots}{N!\times(M-N)\big(M-(N+1)\big)\dots}$$

$$\Omega = \frac{M!}{N!\,(M-N)!}$$

Als M heel groot is, dan zullen de termen $(M-1)$ t/m $\big(M-(N-1)\big)$ niet veel van M verschillen en geldt bij benadering

$$\Omega = \frac{M(M-1)(M-2)\dots\big(M-(N-1)\big)}{N!} \approx \frac{M\times M\times M\times\dots\times M}{N!} = \frac{M^N}{N!}$$

(b) De entropie is gegeven door de Boltzmann-formule als evenredig met het aantal realiseringsmogelijkheden. Vergelijking van de entropie voor hetzelfde systeem bij twee volumes levert dan dat het entropieverschil is gegeven door:

$$\Delta S = S(nM) - S(M) = k\ln\Omega(nM) - k\ln\Omega(M) = k\ln\tfrac{\Omega(nM)}{\Omega(M)}$$

$$\Delta S = k\ln\left[\frac{(nM)^N}{N!}\times\frac{N!}{M^N}\right] = k\ln\frac{(nM)^N}{M^N} = Nk\ln n$$

Hierbij is het tweede volume n maal zo groot genomen als het eerste volume. Het entropieverschil vertoont een logaritmisch verband met de schalingsfactor n. Als het volume n maal zo groot wordt, neemt de entropie een factor $\ln n$ toe, ofwel de entropie is evenredig met de logaritme van het volume.

(c) Het entropieverschil volgt uit het toepassen van de Boltzmann formule voor de beide situaties. Bedenk dat voor de beginsituatie, de gescheiden systemen, de entropieën moeten worden opgeteld.

$$\Delta S = S(gemengd) - S(ongemengd) = k\ln\Omega(gemengd) - k\ln\Omega(ongemengd)$$

$$\Delta S = k\ln\frac{\Omega(gemengd)}{\Omega(ongemengd)} = k\ln\left[\frac{(N_1+N_2)^{N+M}}{N!\times M!}\times\frac{N!}{N_1^N}\times\frac{M!}{N_2^M}\right]$$

$$\Delta S = k\ln\frac{(N_1+N_2)^N\times(N_1+N_2)^M}{N_1^N\times N_2^M} = kN\ln\frac{(N_1+N_2)}{N_1} + kM\ln\frac{(N_1+N_2)}{N_2} > 0$$

De formule voor de gemengde situatie volgt uit dezelfde redenering als bij deelvraag 1: alle volume-eenheden staan alle deeltjes ter beschikking. Bij het corrigeren voor identieke deeltjes moet echter rekening gehouden worden dat A niet identiek is met B, vandaar het product $N!\times M!$. De faculteiten vallen tegen elkaar weg en om snel in te kunnen zien dat de entropieverandering altijd positief is kunnen de machten in N en M het beste worden gescheiden. Omdat $N_1+N_2 > N_1$ en $N_1+N_2 > N_2$, is het argument van de beide ln-termen > 1 en zijn de ln-termen > 0.